Trigonometrik İntegraller:

İntegrallerin, trigonometrik fonksiyonların cebirsel kombinasyonları olan integrallerdir. Bunların başlıcaları aşağıda verilmektedir.

1) $\int \sin ax \sin bx \, dx$, $\int \sin ax \cos bx \, dx$

$\int \cos ax \cos bx \, dx$ tipindeki integrallerdir.

$\cos(a+b)x = \cos ax \cos bx - \sin ax \sin bx$ ... (x)

$\cos(a-b)x = \cos ax \cos bx + \sin ax \sin bx$ ... (xx)

(x) ve (xx) taraf tarafa toplanırsa,

$2\cos ax \cos bx = \cos(a+b)x + \cos(a-b)x$

$\cos ax \cos bx = \frac{1}{2}\left[\cos(a+b)x + \cos(a-b)x\right]$

$\int \cos ax \cos bx \, dx = \frac{1}{2}\int \left[\cos(a+b)x + \cos(a-b)x\right] dx$

$= \frac{1}{2}\left[\frac{\sin(a+b)x}{a+b} + \frac{\sin(a-b)x}{a-b}\right] + c$

Ör. $\int \cos 2x \cos 4x \, dx = \frac{1}{2}\left[\frac{\sin 6x}{6} + \frac{\sin(-2x)}{-2}\right] + c$

$a=2$ $b=4$

$= \frac{1}{2}\left(\frac{\sin 6x}{6} - \frac{\sin 2x}{-2}\right) + c$

$\cos(a+b)x$
$\cos(a-b)x$

(x) ve (xx) taraf tarafa çıkarılırsa,

$$\cos(a+b)x - \cos(a-b)x = -2 \cdot \sin ax \sin bx$$

$$\sin ax \sin bx = \frac{-1}{2}\left[\cos(a+b)x - \cos(a-b)x\right]$$

$$\int \sin ax \sin bx \, dx = \frac{-1}{2}\int\left(\cos(a+b)x - \cos(a-b)x\right)dx$$

$$= \frac{-1}{2}\left[\frac{\sin(a+b)x}{a+b} - \frac{\sin(a-b)x}{a-b}\right] + c$$

$$\int \sin \underset{\substack{\uparrow \\ a=3}}{3x} \sin \underset{\substack{\uparrow \\ b=5}}{5x} \, dx = \frac{-1}{2}\left[\frac{\sin 8x}{8} - \frac{\sin(-2x)}{-2}\right] + c$$

$$= \frac{-1}{2}\left[\frac{\sin 8x}{8} - \frac{\sin 2x}{2}\right] + c$$

$$\sin(a+b)x = \sin(ax+bx) = \sin ax \cos bx + \cos ax \sin bx \quad --- \text{(x)}$$

$$\sin(a-b)x = \sin(ax-bx) = \sin ax \cos bx - \cos ax \sin bx \quad --- \text{(xx)}$$

(x) ve (xx) taraf tarafa toplanırsa,

$$\sin(a+b)x + \sin(a-b)x = 2 \cdot \sin ax \cos bx$$

$$\sin ax \cos bx = \frac{1}{2}\left[\sin(a+b)x + \sin(a-b)x\right]$$

$$\int \sin ax \cos bx = \frac{1}{2}\int\left[\sin(a+b)x + \sin(a-b)x\right]dx$$

ÖR: $\int \sin x \cos 4x \, dx = \frac{1}{2}\int \big(\sin 5x + \underbrace{\sin(-3x)}_{-\sin 3x}\big)\, dx$

$= \frac{1}{2} \cdot \int \sin 5x \, dx - \frac{1}{2}\int \sin 3x \, dx$

$= \frac{-1}{2} \frac{\cos 5x}{5} + \frac{1}{2} \frac{\cos 3x}{3} + c$

ÖR:

2) $\int \sin^m x \cos^n x \, dx$ tipindeki integraller,

m ve n'nin durumuna göre, farklı değişken değiştirme yardımıyla çözülür.

i) m tek ise $\Rightarrow \cos x = u$

n tek ise $\Rightarrow \sin x = u$

değişken değiştirmesi yapılarak integral hesaplanması yapılır.

ÖR: $\int \sin^5 x \cdot \cos^2 x \, dx = ?$

m=5 tek olduğundan,

$\cos x = u$

$-\sin x \, dx = du$

$\sin x \, dx = -du$

$I = \int \sin^4 x \cdot \cos^2 x \cdot \underbrace{\sin x dx}_{-du}$

$I = \int (1-\cos^2 x)^2 \cdot \cos^2 x \sin x \, dx$

$I = -\int (1-u^2)^2 \cdot u^2 \, du$

$-\int u^2 (1 - 2u^2 + u^4) \, du$

$I = -\int u^2 - 2u^4 + u^6 \, du$

$I = -\left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + c$

$u = \cos x$

Ör: $\int \sin^4 x \cdot \cos^3 x \, dx = ?$

$\sin x = u$ dön. yapılır

$\cos x \, dx = du$

$$I = \int \sin^4 x \cdot \cos^2 x \cos x \, dx$$

$$= \int \sin^4 x \cdot (1 - \sin^2 x) \cdot du$$

$$= \int u^4 \cdot (1 - u^2) \, du$$

$$= \int u^4 - u^6 \, du \Rightarrow \frac{u^5}{5} - \frac{u^7}{7} + c$$

$u = \sin x$ yaz.

Ör: $\int \sin^3 x \cdot \cos^3 x \, dx = ?$

Herhangi birine u de. ikisi de tek

$\sin x = u \lor \cos x = u$ denilebilir.

$\cos x \, dx = du$

$$I = \int \sin^3 x \cdot \cos^2 x \cdot \cos x \, dx$$

$$= \int \sin^3 x \cdot (1 - \sin^2 x) \cos x \, dx$$

$$\int u^3(1-u^2)\,du \Rightarrow \int u^3\,du - \int u^5\,du$$

$$= \frac{u^4}{4} - \frac{u^6}{6} \Big|_{u=\sin x} \text{ yaz.}$$

(4) $= \int \sin^5 x \, dx = ?$ $\left(\sin^5 x \cdot \cos^0 x\right)$

$\cos x = u$

$\sin x\,dx = -du$

$$I = \int \underbrace{\sin^4 x}\,\sin x \, dx \Rightarrow \int \underbrace{(\sin^2 x)}^2 \cdot \sin x\,dx$$

$$\int (1-\cos^2 x)^2 \cdot \sin x\,dx \Rightarrow -\int (1-u^2)^2 \cdot du$$

$$= -\int (1-2u^2+u^4)\,du$$

$$= -u + \frac{2u^3}{3} - \frac{u^5}{5} + c \Big|_{u=\cos x} \text{ yaz}$$

(ii) $\int \sin^m x \cos^n x \, dx$

m ve n çift ise

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

Q: $\int \sin^4 x \cos^2 x \, dx = ?$

$$\int (\sin^2 x)^2 \cdot \cos^2 x \, dx$$

$$\int \left(\frac{1-\cos 2x}{2}\right)^2 \cdot \left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{8}\int (1-2\cos 2x+\cos^2 2x)(1+\cos 2x)\,dx$$

$$\frac{1}{8}\int (1+\cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x)\,dx$$

$$= \frac{1}{8}\int (1-\cos 2x - \cos^2 2x + \cos^3 2x)\,dx$$

tek

$$= \frac{1}{8}\left[\int dx - \int \cos 2x\,dx - \int \cos^2 2x + \int \cos^3 2x\,dx\right.$$

$$= \frac{1}{8}\; x - \frac{\sin 2x}{2} - \int \frac{1+\cos 4x}{2}\,dx + \int \cos^3 2x\,dx$$

$$= \frac{1}{8}x - \frac{\sin 2x}{2} - \frac{x}{2} - \frac{1}{2}\,\frac{\sin 4x}{4} + \int \cos^3 2x\,dx$$

m=3 tek

$$\int \cos^3 2x\,dx = \int \cos^2 2x \cdot \cos 2x\,dx$$

$\sin 2x = u$

$2\cos 2x\,dx = du$

$$= (1-\sin^2 2x)\cos 2x\,dx$$

$$= \frac{1}{2}\int (1-u^2)\,du$$

$$\frac{1}{2}u - \frac{1}{2}\frac{u^3}{3} + c.$$

$u = \sin 2x$ yaz

Örnek: $\displaystyle\int \frac{\sqrt{x^2+x}}{x}\,dx = ?$

Ör: $\displaystyle\int \frac{x^2}{\sqrt{16x^2+9}}\,dx = ?$

$$\int \frac{x^2}{\sqrt{(4x)^2+3^2}}$$

x^2+a^2

$\sqrt{(bx)^2+a^2}$; $bx = a.\tan t$

$4x = 3.\tan t$ dön. yapılır

$$x = \frac{3}{4}\tan t$$

$$dx = \frac{3}{4\cos^2 t}\,dt$$

$$\int \frac{x^2+x}{x\sqrt{x^2+x}}\,dx = \int \frac{x+1}{\sqrt{x^2+x}}\,dx$$

$$= A\cdot\sqrt{x^2+x} + \lambda\int \frac{dx}{\sqrt{x^2+x}}$$

$$\underset{(2)}{\frac{x+1}{\sqrt{x^2+x}}} = \frac{2x+1}{2\sqrt{x^2+x}}A + \underset{(2)}{\frac{A}{\sqrt{x^2+x}}}$$

$2x+2 = 2$

$$I = \int \frac{\frac{9}{16}\,\frac{\sin^2 t}{\cos^2 t}}{\sqrt{\underbrace{9\tan^2 t+9}_{9\left(\frac{1}{\cos^2 t}\right)}}}\cdot\frac{3}{4\cos^2 t}\,dt = \int \frac{\frac{9}{16}\cdot\frac{\sin^2 t}{\cos^2 t}}{\frac{\cancel{3}}{\cancel{\cos t}}}\cdot\frac{\cancel{3}}{4}\cdot\frac{1}{\cos^2 t}\,dt$$

$$= \frac{9}{64}\int \frac{\sin^2 t}{\cos^3 t}\,dt = \frac{9}{64}\int \frac{1-\cos^2 t}{\cos^3 t}\,dt$$

$$= \frac{9}{64}\underbrace{\int \frac{dt}{\cos^3 t}}_{I_1} - \frac{9}{64}\underbrace{\int \frac{dt}{\cos t}}_{I_2}$$

$$I_1 = \int \underbrace{\frac{1}{\cos t}}_{u} \cdot \underbrace{\frac{1}{\cos^2 t}}_{dv} \, dt$$

~~2. yol~~ $(Ax+B) \cdot \sqrt{16x^2+9} + \lambda \int \frac{1}{\sqrt{16x^2+9}} dk$

$$\frac{1}{\cos t} = u \qquad \int \frac{1}{\cos^2 t} dt = \int dv$$

$$\frac{\sin t \, dt}{\cos^2 t} = du \qquad \tan t = v$$

$$\frac{\tan t}{\cos t} - \int \frac{\sin t}{\cos t} \cdot \frac{\sin t}{\cos^2 t} dt$$

$$= \frac{\tan t}{\cos t} - \int \frac{(1-\cos^2 t)}{\cos^3 t} dt = \frac{\tan t}{\cos t} - \int \frac{dt}{\cos^3 t} + \int \frac{1}{\cos t} dt$$

$$① \int \frac{1}{\cos t} dt \Rightarrow \int \frac{\cos t}{\cos^2 t} dt = \int \frac{\cos t}{1-\sin^2 t} dt = \int \frac{dw}{1-w^2} = \frac{-1}{2} \ln \left| \frac{1-w}{1+w} \right|$$

$\sin t = w$

$$= \frac{-1}{2} \ln \left| \frac{1-\sin t}{1+\sin t} \right| + C$$

$$I_1 = \frac{\tan t}{\cos t} - I_1 - \frac{1}{2} \ln \left| \frac{1-\sin t}{1+\sin t} \right|$$

$$2I_1 = \frac{\tan t}{\cos t} - \frac{1}{2} \ln \left| \frac{1-\sin t}{1+\sin t} \right| \rightarrow I_1 = \frac{1}{2} \left(\frac{\tan t}{\cos t} - \frac{1}{2} \ln \left| \frac{1-\sin t}{1+\sin t} \right| \right)$$

42. $\int \frac{\sin 2x}{1+\sin^2 x}\,dx = ?$

$\tan x = u$ ✓

$\sin^2 x = w$

$\sin x = w$

$\cos\,dx = dw$

$$I = \int \frac{2\sin x \cdot \cos x}{1+\sin^2 x}\,dx = \int \frac{2 \cdot w \cdot dw}{1+w^2} = \int \frac{du}{u}$$

$1+w^2 = u$

$2w\,dw = du$

$\ln|u| + c$

$u = \sin x$ yol.

43. $\int \frac{x^3\,dx}{\sqrt{1-x^2}} = ?$

1. yol → $x = a.\sin t$) $\sqrt{a^2-x^2}$

$dx = a.\cos t$

2. yol → $Q_{n-1}(x)\sqrt{ax^2+bx+c} + \lambda \int \frac{dx}{\sqrt{ax^2+bx+c}}$

$u = \frac{Ax+B}{1+x^2} + \frac{Cx+D}{(1+x^2)^2}$

$$I = \int \frac{\sin^3 t \cdot \cancel{\cos t}}{\sqrt{1-\sin^2 t}}\,dt = \int \sin^3 t\,dt$$

$ax^2+bx+c\sqrt{1-x^2} + \lambda \int \frac{dx}{\sqrt{1-x^2}}$

$\int \sin^2 t \cdot \sin t\,dt$

$(1-\cos^2 t)\sin t\,dt$

$\cos t = w \qquad -\sin t\,dt = dw$

$= (1-w^2)\,dw$

$= -w + \frac{w^3}{3} + c = -\cos t + \frac{\cos^3 t}{3} + c$

$t = \arcsin x$ yaz.

$$\frac{x^2+1}{x^4+1} = \frac{Ax+B}{x^2+1-\sqrt{2}x} + \frac{Cx+D}{x^2+1+\sqrt{2}x}$$

ÖR: $\displaystyle\int \frac{x^2+1}{x^4+1}\,dx = ?$

$(x^2+1)^2 \rightarrow x^4+2x^2+1 - 2x^2$

$(x^2+1)^2 - (\sqrt{2}x)^2$

1. yol: $\left(x-\frac{1}{x}\right) = u \quad \longrightarrow \quad x^2 - 2 + \frac{1}{x^2} = u^2$

$1+\frac{1}{x^2}\,dx = du \qquad x^2+\frac{1}{x^2} = u^2+2$

$\frac{x^2+1}{x^2}\,dx = du \qquad \frac{x^4+1}{x^2} = u^2+2$

$$I = \int \frac{\frac{x^2+1}{x^2}\,dx}{\frac{x^4+1}{x^2}} = \int \frac{du}{(\sqrt{2})^2+u^2} = \frac{1}{\sqrt{2}}\arctan\left(\frac{u}{\sqrt{2}}\right)+c$$

$u = x - \frac{1}{x}$

2. yol: $x^4+1 = (x^2+1)^2 - 2x^2 = (x^2+1-\sqrt{2}x)(x^2+1+\sqrt{2}x)$

$$\frac{x^2+1}{x^4+1} = \frac{Ax+B}{x^2+1-\sqrt{2}x} + \frac{Cx+D}{x^2+1+\sqrt{2}x}$$

$(x^2+1+\sqrt{2}x) \quad (x^2+1-\sqrt{2}x)$

$x^2+1 = (Ax+B)(x^2+1+\sqrt{2}x) + (Cx+D)(x^2+1-\sqrt{2}x)$

ÖR:

$Ax^3 + \cancel{Ax} + A\sqrt{2}x^2 + \cancel{Bx^2} + B + \cancel{B\sqrt{2}x} + Cx^3 + \cancel{Cx} - \cancel{C\sqrt{2}x^2}$

$+ \cancel{Dx^2} + D - D\sqrt{2}x = x^2+1$

$A+C=0 \qquad A\sqrt{2} - C\sqrt{2} + \underbrace{B+D}_{1} = 1$

$A - D\sqrt{2} + B\sqrt{2} + C = 0 \qquad B+D = 1$

$\sqrt{2}(B-D) = 0 \qquad \sqrt{2}(A-C) = 0$

$$\begin{aligned} B-D&=0 \\ B+D&=1 \\ \hline 2B&=1 \end{aligned} \qquad \begin{aligned} A-C&=0 \\ A+C&=0 \\ \hline A&=0 \\ C&=0 \end{aligned}$$

$B = \frac{1}{2}$

$D = \frac{1}{2}$

11) $\int t^3 \cdot \arctan t \, dt = ?$

LAP TU

$\arctan t \quad t^3$

$\arctan t = u \qquad \int t^3 \, dt = \int dv$

$\frac{1}{1+t^2} dt = du \qquad \frac{t^4}{4} = v$

$$\arctan t \cdot \frac{t^4}{4} - \int \frac{t^4}{4} \cdot \frac{dt}{1+t^2}$$

$$\arctan t \cdot \frac{t^4}{4} - \frac{1}{4} \int \frac{t^4}{1+t^2}$$

$$\begin{array}{r|l} t^4 & t^2+1 \\ \hline -t^4 + t^2 & t^2 - 1 \\ \hline -t^2 & \\ t^2+1 & \\ \hline 1 & \end{array}$$

$$\arctan t \cdot \frac{t^4}{4} - \frac{1}{4} \int \left(t^2 - 1 + \frac{1}{t^2+1} \right) dt$$

$$\arctan t \cdot \frac{t^4}{4} - \frac{1}{4} \left(\frac{t^3}{3} - t + \arctan t \right) + c$$

$$\int \frac{x^2+1}{x^4+1} \, dx$$

(x^2+1)

$(x^4+1) = x^4 + 2x^2 + 1 \quad -2x^2$

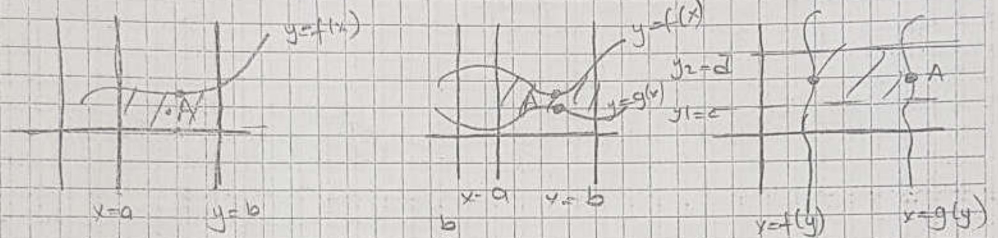
$(x^2+1)^2 - 2x^2$

$(x^2+1-\sqrt{2}x) + (x^2+1+\sqrt{2}x)$

CAPTU

Belirli integral:

* Belirli integral kavramı, $y=f(x)$ eğrisi x ekseni, $x=a$ ve $y=b$ doğruları arasında kalan bölgenin alanını hesaplamak için kullanılan kavramdır. Uygulamalı mat. diğer birçok son probleminde karşımıza çıkar.



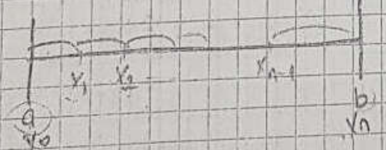
$$A = \int_a^b f(x)\,dx$$

$$A = \int_a^b f(x) - g(x)$$

$$A = \int_c^d (g(y) - f(y))\,dy$$

Aralıkların Parçalanması:

a, b kapalı aralığını $a < x_1 < x_2 < \dots < x_{n-1} < b$ özelliğini sağlayan $x_1, x_2, \dots x_{n-1}$ noktaları yardımıyla n tane alt aralığa bölelim. Genelliği bozmamak için $a = x_0$ ✓ $b = x_n$ ✓ ifade edelim.



$P = \{x_0, x_1, \dots x_{n-1}, x_n\}$ kümesine $[a, b]$ ✓ bölüntüsü veya parçalanması denir.

Dolayısıyla $[a,b]$ şeklinde
$[x_0, x_1]$, $[x_1, x_2]$ ––– $[x_{n-1}, x_n]$ kapalı alt aralıkla-
rına bölünür.

$\Delta x_k = x_k - x_{k-1}$ ile her bir kapalı alt aralığın boyu veya ölçüsü bulunmuş olur. Alt aralıkların en büyüğüne P'nin normu denir. $\|P\|$, ile gösterilir.

$$\|P\| = \max\{\Delta x_1, \Delta x_2 \cdots \Delta x_k\}$$

$\Delta x_1 = \Delta x_2 = \cdots = \Delta x_k$ ise P parçalanmasına düzgün parçalanma denir.

TANIM: $f: [a,b] \to \mathbb{R}$ sürekli bir fonksiyon P ise $[a,b]$ bir bölüntüsü olsun

$$M_k = \max\{f(x) : x_{k-1} \leq x \leq x_k\}$$

$$m_k = \min\{f(x) : x_{k-1} \leq x \leq x_k\}$$ olm. üzere

$A(f,P)$ f fonksiyonunun P parçalanmasına karşılık gelen

$$A(f,P) = \sum_{k=1}^{n} m_k \cdot \Delta x_k \quad , \quad Ü(f,P) = \sum_{k=1}^{n} M_k \cdot \Delta x_k$$

olarak sırasıyla alt Darboux toplamı üst Darboux toplamı elde edilir.

$\forall x_k^* \in [x_{k-1}, x_k]$ için

$$R(f,P) = \sum_{k=1}^{n} f(x_k^*) \cdot \Delta x_k$$ ile f fonk. P parçalanmasına

karşılık gelen Riemann toplamı denir. ve açıktır ki,

$A(f,P) \leq R(f,P) \leq \ddot{U}(f,P)$

aralıkların yeterince küçük seçilmesiyle yani 0'a yaklaştıkça

$\|P\| \to 0 \quad (n \to \infty)$

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k = \int_a^b f(x).dx \text{ 'dir}$$

Ör: $\int_0^1 x^2\,dx$ tanım yardımıyla çöz.

fonk. yan

$M_k = \max\{f(x) : x_{k-1} \leq x \leq x_k\} = f(x_k) = f\left(\frac{k}{n}\right) = \left(\frac{k}{n}\right)^2$

$m_k = \min\{f(x) : x_{k-1} \leq x \leq x_k\} = f(x_{k-1}) = f\left(\frac{k-1}{n}\right) = \left(\frac{k-1}{n}\right)^2$

$[a,b] = [0,1] \qquad \Delta x_k = \frac{1-0}{n} = \frac{1}{n}$

$x_0=0 \quad \underbrace{x_1=1/n}_{1/n} \quad \underbrace{x_2=2/n}_{1/n} \quad \ldots \quad \frac{k-1}{n} \quad \frac{k}{n} \quad \ldots \quad x_n = \frac{n}{n}$

$P = \{0, x_1, x_2, \ldots, x_{k-1}, x_k, 1\}$

$= \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{k-1}{n}, \frac{k}{n}, 1\}$

$\ddot{U}(f,P) = \sum_{k=1}^{n} M_k\,\Delta x_k = \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 \cdot \frac{1}{n} \Rightarrow \frac{1}{n^3} \sum_{k=1}^{n} k^2$

$\ddot{U}(f,P) = \frac{1}{n^3}\left(1^2+2^2+\ldots+n^2\right) = \frac{1}{n^3} \cdot \frac{n.(n+1)(2n+1)}{6}$

$A(f,P) = \sum_{k=1}^{n} m_k\,\Delta x_k = \sum_{k=1}^{n} \left(\frac{k-1}{n}\right)^2 \cdot \frac{1}{n} = \frac{1}{n^3} \cdot \sum_{k=1}^{n} (k-1)^2$

$A(f,P) = \frac{1}{n^3} \cdot \frac{(n-1) \cdot n \cdot (2n-1)}{6}$

$\|P\| \to 0 \quad (n \to \infty)$

$(\|P\| \to 0) \quad \lim_{n \to \infty} \frac{n(n+1)(2n+1)}{6n^3} = \lim_{n \to \infty} \frac{(n-1)(n)(2n-1)}{6n^3} = \int_0^1 x^2 \, dx$

$\frac{1}{3} = \int_0^1 x^2 \, dx$ ✓

Integral

$f: [a,b]$ sürekli bir fonksiyon P ise bölüntü olmak üzere $F'(x) = f(x)$ olacak şekilde, bir F fonksiyonu var ise,

$\{F : [a,b] \to \mathbb{R}\}$

$\int_a^b f(x)\,dx = F(b)$'i $- F(a)$'ı şeklinde bulunur. ✓

Bu kurala (Newton – Leibnitz) denir.

$\int_{a=0}^{b=1} \underbrace{\frac{2x}{1}}_{f(x)} dx = x^2 \Big|_0^1 = 1^2 - 0^2 = 1$

$F(1) - F(0)$

NOT: f, $[a,b]$ integrallenebilir bir fonksiyon için,

(i) $\int_a^a f(x)\,dx = 0$

(ii) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

(iii) $\int_a^b f(x)\,dx = \int_a^b f(w)\,dw = \int_a^b f(t)\,dt$

(iv) $\forall c \in (a,b)$ için,

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

NOT: f, $[a,b]$ integrallenebilir bir fonksiyon olmak üzere,

(i) f tek fonksiyon ise, ($f(-x) = -f(x)$ ise)

$\int_{-a}^{a} f(x)\,dx = 0$ dır.

Ör: $\displaystyle\int_{-\pi}^{\pi} \underbrace{\frac{x^3 \cdot \cos x}{1+\sin^{10}x}}_{f(x)}\,dx = ?$

$$f(-x) = \frac{(-x)^3 \cdot \cos(-x)}{1+\underbrace{(\sin(-x))^{10}}_{(-\sin x)^{10}}} = \frac{-x^3 \cdot \cos x}{1+\sin x^{10}} = -f(x)$$

$I = 0$ ✓

(ii) f çift fonksiyon ise, ($f(-x) = f(x)$ ise

$$\int_{-a}^{a} f(x)\,dx = 2 \cdot \int_0^a f(x)\,dx$$

$\int_a^b f(x)\,dx$ hesaplamak için $F'(x) = f(x)$ olan bulunuyorsa Newton - Leibnitz kuralından,

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

idi. Fakat türevi $f(x)$ $F(x)$ fonk. bulmak her zaman kolay olmayabilir. Belirsiz integralde verilen değişken değiştirme ve kısmi integrasyon formülleri belirli integraller için geçerlidir. Belirli integralde değişken değiştirmesi yapıldığında sınırlar aynı kalmaz. Yani

$g(x) = u \quad g'(x) = du$

$$\int_a^b f(g(x)) \cdot g'(x) \cdot dx = \int_{g(a)}^{g(b)} f(u)\,du$$

şeklinde bulunur.

Benzer şekilde, belirli integraller için kısmi integrasyon yöntemi,

$$\int_{x=a}^{x=b} u(x)\, d(v(x)) = u(x) \cdot v(x) \Big|_a^b - \int_a^b v(x) \cdot d(u(x))$$

şeklinde bulunur.

①: $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\, dx = ?$

$\sin x = u$

$\cos x\, dx = du$

$I = \int_\alpha^\beta \frac{du}{1+u^2} = \arctan u \Big|_\alpha^\beta$

$= \arctan(\sin x) \Big|_0^{\pi/2}$

$\arctan(\underbrace{\sin \pi/2}_{1}) - \underbrace{\arctan 0}_{0}$

$= \pi/4$

②: $\int_0^\pi \underset{P}{x} \cdot \underset{T}{\cos x}\, dx = ?$

$x = u$ $\quad \int \cos x\, dx = \int dv$

$dx = du$ $\quad \sin x = v$

$I = u \cdot v \Big|_0^\pi - \int_0^\pi v \cdot du = x \cdot \sin x \Big|_0^\pi - \sin x \cdot dx$

$\underbrace{\pi \cdot \sin \pi}_{-\pi} - \cancel{0 \cdot \sin 0} - \int_0^\pi \sin x\, dx = \cos x \Big|_0^\pi$

$= \underset{0}{\cos \pi} - \underset{1}{\cos 0}$

$= -1 - 1 = -2$?

$-\pi - 1$?

Teo: (Leibnitz formülü) (İntegrallerin türevlendirilmesi)

$f, [a,b]$ integrallenebilen bir fonk. olsun.

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)\,dt = f(v(x))\cdot v'(x) - f(u(x))\cdot u'(x)$$

Ör: $F(x) = \int_{x^2}^{x^3} \sin(t^2)\,dt$ ise $F'(x) = ?$

$$F'(x) = \frac{d}{dx}\int_{x^2}^{x^3} \sin(t^2)\,dt = \sin(x^6)\cdot 3x^2 - \sin(x^4)\,2x$$

Ör: $\lim\limits_{x\to 0} \frac{1}{x^2}\int_0^x \frac{t^2}{t^4+1}\,dt = ?$ $(0\cdot\infty)$ $\left(\frac{0}{0} \vee \frac{\infty}{\infty}\right)$'a Çevir

$$= \lim_{x\to 0} \frac{\int_0^x \frac{t^2}{t^4+1}dt}{x^2} \stackrel{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{x^2\cdot 1 - 0}{1+x^4}}{2x}$$

$$\lim_{x\to 0} \frac{x}{1+x^4} = 0 \checkmark$$

İntegrallerin Uygulanması:

1) Alan Hesabı: f $[a,b]$ sürekli bir fonk. olsun. $y = f(x)$ fonk. $x=a$, $x=b$ doğruları ile x ekseni arasındaki bölgenin alanını hesaplayalım. Bu alan $[a,b]$ aralığında $f(x)$ fonk. işaretine göre değişik biçimlerde hesaplanır.

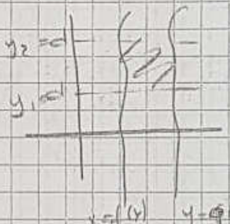
1) $[a,b]$ $f(x) \geq 0$ olsun.

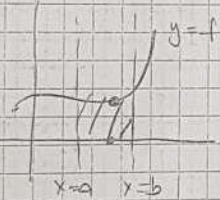
$[a,b]$ P- bölüntüsünü $P = \{x_0, x_1, \dots, x_{n-1}, x_n\}$ ile gösterelim

$x_k^* \in [x_{k-1}, x_k]$ olsun. Hesaplanması istenen A alanı yaklaşık olarak eni Δx ve boyu $f(x_k^*)$ olan dik dörtgenlerin alanları toplamına eşittir.

$$A \cong \sum_{k=1}^{n} f(x_k^*) \cdot \Delta x_k$$

$$A \cong \lim_{\|P\| \to 0} \sum_{k=1}^{n} f(x_k^*) \cdot \Delta x_k$$

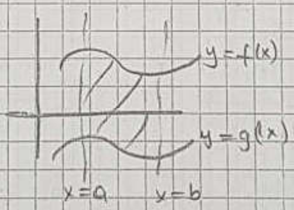


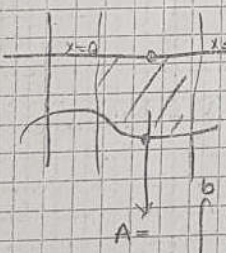


$$A = \int_a^b f(x)\,dx$$

$x=f(y)$ $y=g(y)$

$$A = \int_c^d g(y) - f(y)\,dy$$

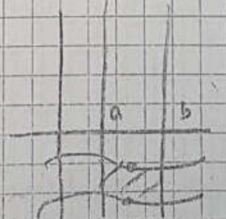




$$A = \int_a^b (0 - f(x))\,dx$$

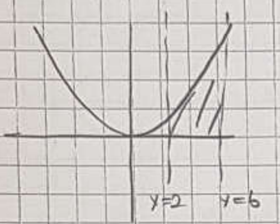
$$= -\int_a^b f(x)\,dx$$

$$A = \int_a^b f(x) - g(x)\,dx$$



$$A = \int_a^b (f(x) - g(x))\,dx$$

Ör: $y=x^2$ eğrisi $x=2$ $x=6$ doğruları ile x ekseni arasında kalan eğri?



$$A=\int_2^6 x^2\,dx \qquad \left.\frac{x^3}{3}\right|_2^6$$

$$\frac{1}{3}\left(6^3-2^3\right) = 63,$$

Ör: $y=x^2-4x+3$ eğrisi $x=2$ $x=4$ ile x ekseni arasında kalan bölgenin alanı?

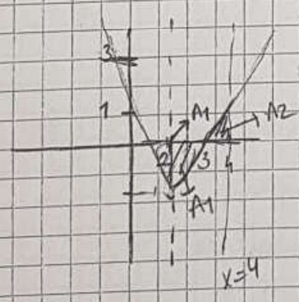
$y=(x-3)\,(x-1)$

$y=0$ $x=3$ $(3,0)$

$x=0$ için $(0,3)$ $(0,1)$

$T(r,k)$ $r=\frac{-b}{2a}=\frac{4}{2}=2$ $\Big\}\,(2,-1)$

$y=4-8+3=-1$



$$A=A_1+A_2=\int_2^3 0-(x^2-4x+3)\,dx+\int_3^4 (x^2-4x+3-0)\,dx$$

$$=\left.\left(-\frac{x^3}{3}+\frac{4x^2}{2}-3x\right)\right|_2^3+\left.\left(\frac{x^3}{3}-\frac{4x^2}{2}+3x\right)\right|_3^4$$

$$=(-9+18-9)-\left(-\frac{8}{3}+8-6\right)+\left(\frac{64}{3}-32+12\right)$$

$$-(9-18+9)$$

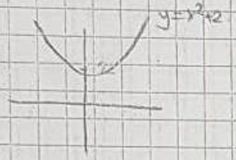
$$=\frac{8}{3}-8+6+\frac{64}{3}-32+12$$

$$=2\ \checkmark$$

(4) $\mathcal{B}$ = $y = x^2+2$, $y = 2x - x^2$ Oy $x=3$ bölgeleri arasındaki kalan bölgenin alanını bulunuz.

$y = x^2+2$ $x=0$ için $y=2$

$y=0$ için $x^2+2=0$
x yok.



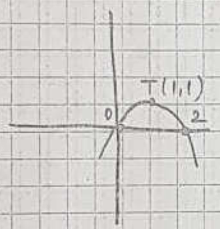
$y = 2x - x^2 = x(2-x)$ $x=0$ için $y=0$

$y=0$ için $x=0$, $x=2$

$x_0 = \frac{-b}{2a} = \frac{-2}{-2} = 1$

$y_0 = 1$

$T(1,1)$



$x^2+2 = 2x - x^2$

$2x^2 - 2x + 2 = 0$

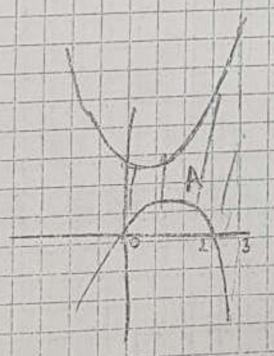
$x^2 - x + 1 = 0$

$\Delta = b^2 - 4ac = 0$ $(-1)^2 - 4 \cdot 1 \cdot 1$

$= 1 - 4 = -3 < 0$

real kök yok.

Yani eğriler kesişmez.



$A = \int_0^3 \left(x^2+2 - (2x - x^2)\right) dx$

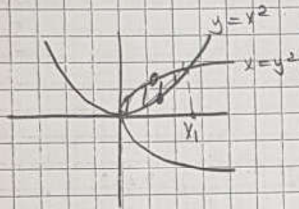
$\int_0^3 (x^2+2-2x+x^2)\,dx$

$$\frac{2x^3}{3} + 2x - x^2 \Big|_0^3 = \frac{2}{3} \cdot 3^3 + 2 \cdot 3 - 3^2$$

$$= 18 + 6 - 9$$

$$= 15$$

12: $y = x^2$ ve $x = y^2$ par. arasında kalan bölgenin alanını bulunuz



$\Rightarrow \boxed{y = \sqrt{x}}$

$y = x^2 \quad x = y^2$ kesişim noktası:

$$x^2 = \sqrt{x}$$

$$x^4 = x$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$x = 0 \wedge x = 1$$

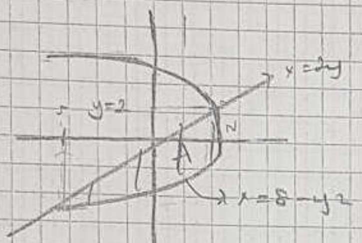
$$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{3}{2}} \Big|_0^1 - \frac{x^3}{3} \Big|_0^1$$

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

(4) D : $x=2y$ doğrusu ile $x=8-y^2$ eğrisi arasında kalan bölgenin alanını bulunuz.

y'nin değerine göre x artıyor.



$y=0$ için $x=8$

$x=0$ için $y^2=8$ $y_{1,2}=\mp 2\sqrt{2}$

$2y=8-y^2$

y^2+2y-8

$y \quad 4$

$y \quad -2$

$y=-4 \quad y=2$

$$A=\int_{y=-4}^{y=2}(8-y^2-2y)\,dy$$

$$=8y-\frac{y^3}{3}-y^2\Bigg|_{y=-4}^{y=2}=36$$

(5) : $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}\,dx$

$$I=\int \frac{2\sin x\cdot\cos x}{\sqrt{9-(\cos^2 x)^2}}\,dx=\int\frac{-du}{\sqrt{9-u^2}} \Rightarrow -\arcsin\left(\frac{u}{3}\right)+c\,\Big|_{u=\cos^2 x}$$

$\cos^2 x=u$

$-2\cos x\,\sin x\,dx=du$

2. yol

$\cos^2 x=3\sin t$

(11)e : $\int \frac{dx}{(x+1)^3\sqrt{(x^2+2x)}}$

$\frac{1}{x+1} = u$ $\quad$ $x+1 = \frac{1}{u}$

$x = \frac{1}{u} - 1 = \frac{1-u}{u}$

$dx = \frac{-1}{u^2}\,du$

$$\frac{\frac{-1}{u^2}\,du}{\frac{1}{u^3} \cdot 1-u}$$

2 yol : $\frac{dx}{(x+1)^3\sqrt{(x+1)^2 - 1}}$

$\int \frac{\sin 2x}{\sqrt{3^2-(\cos^2 x)^2}}\,dx$ $\quad$ $\cos^2 x = u$

$-2\cos x \sin x\, dx = du$

$-\sin 2x\, dx = du$

$-\int \frac{du}{\sqrt{3^2 - u^2}} \rightarrow -\arcsin \frac{u}{3}$

$x = a \cdot \sin t$

$\cos^2 x = 3 \cdot \sin t$

(11)R: $\int \frac{3\sin x - 7\cos x}{\sin x + \cos x}\,dx$ 　　 1. yol = $\left[\tan\frac{x}{2} = u\right]$

$3\sin x - 7\cos x = a.(\sin x + \cos x) + b.(\cos x - \sin x)$

$3\sin x - 7\cos x = \sin x\,\underbrace{(a-b)}_{3} + \cos x\,\underbrace{(a+b)}_{-7}$

$a - b = 3$
$a + b = -7$

$2a = -4$
$a = -2$
$b = -5$

$= \int \frac{-2(\sin x + \cos x) - 5(\cos x - \sin x)}{\sin x + \cos x}\,dx$

$\int \frac{-2\cancel{(\sin x + \cos x)}}{\cancel{\sin x + \cos x}}\,dx + \int \frac{-5(\cos x - \sin x)}{\sin x + \cos x}\,dx$

$\left[\sin x + \cos x = w\right.$

$\left.(\cos x - \sin x)\,dx = dw\right]$

$-2\int dx + -5\int \frac{dw}{w}$

$= -2x - 5\,\ln|\sin x + \cos x| + c$

(1)R: $\int \frac{x^4 - 2}{\sqrt{x^2+1}}\,dx = ?$

$I = \int \frac{x^4 - 2}{\sqrt{x^2+1}}\,dx = (ax^3 + bx^2 + cx + d).\sqrt{x^2+1}\,\lambda + \int \frac{dx}{\sqrt{x^2+1}}$

$\frac{x^4 - 2}{\sqrt{x^2+1}} = \frac{3ax^2 + 2bx + c.\sqrt{x^2+1}}{\frac{1}{\sqrt{x^2+1}}} + \frac{2x}{2\sqrt{x^2+1}}.(ax^3 + bx^2 + cx + d) + \frac{\lambda}{\sqrt{x^2+1}}$

$x^4 - 2 = x^2 + 1\,(3ax^2 + 2bx + c).\,x.(ax^3 + bx^2 + cx + d) + \lambda$

$x^4 - 2 = 4ax^4 + 3bx^3 + x^2\,(3a + 2c) + x\,(2b + d)\,[illegible] + c$

$4a = 1$ $\quad a = 1/4$

$3b = 0$ $\quad b = 0$

$3a + 2c = 0$ $\quad 2c = -3a$ $\quad c = \frac{-3}{8}$

$2b + d = 0$ $\quad d = 0$

$\lambda + c = -2$

$\lambda = -2 + \frac{3}{8} = \frac{-13}{8}$ ✓

$$J = \int \frac{x^4 - 2}{\sqrt{x^2+1}}\,dx = \left(\frac{x^3}{4} - \frac{3}{8}x\right)\sqrt{x^2+1} \quad \frac{-13}{8} \int \frac{dx}{\sqrt{x^2+1}}$$

$$= \left(\frac{x^3}{4} - \frac{3}{8}x\right) x^2+1 \quad - \frac{13}{8} \ln\left|x + \sqrt{x^2+1}\right| + c$$

11) S: $\int \sqrt[3]{1+\sqrt[4]{x}}\,dx = ?$ $\qquad x^r \cdot \left((a+b) \cdot x^p\right)^q$

$$\int x^{0} \left(1 + x^{\frac{1}{4}}\right)^{1/3} dx = ?$$

$r = 0 \quad p = \frac{1}{4} \quad q = \frac{1}{3}$

1. durum $\rightarrow$ $q \notin$

2. durum $\rightarrow$ $\frac{r+1}{p} = \frac{1}{\frac{1}{4}} = 4$ ✓ $t \in \mathbb{Z}$ ✓

$$1 + x^{\frac{1}{4}} = w^3$$

$$x = (w^3 - 1)^4$$

$$dx = 4\,(w^3-1)^3 \cdot 3w^2$$

$$dx = 12\,(w^3-1)^3 \cdot w^2$$

$$I = \int \underbrace{(w^3)^{1/3}}_{w \text{ olarak}}\; 12\,(w^3-1)^3 \cdot w^2$$

$$12 \int w \,(w^3-1)^3 \cdot w^2$$

$$12 \int$$

Parametrik Denklemlerle verilen eğrilerin sınırlarının Alan Hesaplanması:

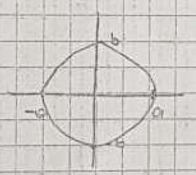
$M(a,b)$ merkezi, r yarıçaplı çember,

$(x-a)^2 + (y-b)^2 = r^2$

Kartezyen denklem (x,y)

$$\begin{cases} x = a + r.\cos\theta \\ y = b + r.\sin\theta \end{cases} = \quad 0 \leq \theta \leq 2\pi$$

$$(\cos\theta)^2 + (r\sin\theta)^2 = r^2 \quad ✓$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{(kartezyen)}$$

$$\begin{cases} x = a.\cos\theta \\ y = b.\sin\theta \end{cases} \quad 0 \leq \theta \leq 2\pi$$

$$\frac{\cancel{a^2}\cos^2\theta}{\cancel{a^2}} + \frac{\cancel{b^2}.\sin^2\theta}{\cancel{b^2}} = 1$$

* f, g t'ye bağlı türevlenebilir iki fonksiyon olsunlar $x = f(t)$ $y = g(t)$ şeklinde t parametresine bağlı bir c eğrisinin $x = a$, $x = b$ doğruları ile x ekseni arasında kalan bölgenin alanını hesaplamak için,

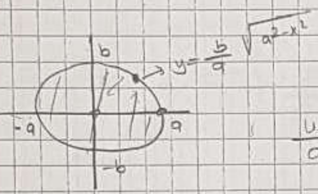
$$A = \int_a^b |y|\, dx$$ alan formülünü t cinsinden hesaplamak mümkündür

$y = g(t)$, $dx = f'(t)\,dt$ olur

t'nin a ve b'ye karşılık gelen değerleri t_1 ve t_2 denirse parametrik şekilde verilmiş eğrinin alanı formülümüz $A = \int_{t_1}^{t_2} |g(t)| \cdot f'(t)\, dt$

(11) R: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ biçiminde verilmiş elips eğrisinin alanı hem kartesyen hem de parametrik şekilde hesaplayınız.

$\dfrac{y^2}{b^2} = 1 - \dfrac{x^2}{a^2} \Rightarrow y^2 = b^2\left(\dfrac{a^2-x^2}{a^2}\right)$



$A = 4\int \dfrac{b}{a}\sqrt{a^2-x^2}\,dx = \dfrac{4b}{a}\int_{x=0}^{x=a} \sqrt{a^2-x^2}\,dx$

$\dfrac{4b}{a}\int_{\alpha}^{\beta} \sqrt{a^2-a^2\sin^2 t}\; a\cos t\, dt = \dfrac{4b}{a}\cdot a^2 \int_{\alpha}^{\beta} \cos^2 t\, dt =$

$x = a\sin t$, dt

$4ab\int_{\alpha}^{\beta} \dfrac{1+\cos 2t}{2}\, dt \Rightarrow 2ab\int_{\alpha}^{\beta} (1+\cos 2t)\, dt$

$\int_{t_1}^{t_2} y(t) \cdot x'(t)\, dt$

$x = a\cos t$
$y = b\sin t$

$= 2ab\left(t + \dfrac{\sin 2t}{2}\right)\Big|_{\alpha}^{\beta}$

$x = 0$ ile $0 = a\sin t$, $\sin t = 0$, $t = 0$

$\beta = \pi/2$

$= 2ab\left(t + \dfrac{\sin 2t}{2}\right)\Big|_{\alpha=0}$

$x = a$ ise $a = a\sin t$, $\sin t = 1$, $t = \pi/2$

$= 2ab\left(\pi/2 + \dfrac{\sin\pi}{2} - 0\right) = \pi \cdot a b$

+ parametrik yazarsak:

$x = a\cos\theta$
$y = b\sin\theta$

$x = 0$ için $0 = a\cos\theta$, $0 = \cos\theta$, $\theta = \pi/2$

$A = \int_{t_1=\pi/2}^{t_2=0} |b\sin\theta| \cdot -a\sin\theta \cdot d\theta$

$x = a$ için $a = a\cos\theta$, $\cos\theta = 1$, $\theta = 0$

$ab\int_0^{\pi/2} \sin^2\theta\, d\theta = ab\int_0^{\pi/2} \dfrac{1-\cos 2\theta}{2}\, d\theta$

$= ab\left(\dfrac{u}{2} - \dfrac{\sin 2\theta}{4}\right)\Big|_0^{\pi/2}$

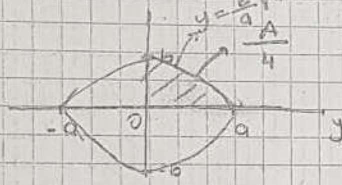
$\cancel{4}\cdot ab\left(\dfrac{\pi}{\cancel{4}}\right)$
$= ab\pi$

(11) Q: $\int \sqrt{(x-1)\cdot(2-x)}\;dx = ?$

$$\int \left[(x-1)\cdot(2-x)\right]^{1/2} dx \qquad \left(x^r\,(a+bx)^p\right)^q$$

(1) Q: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ (Elips denklemi)

Kartezyen hesabı ile:



$$A = 4\int_{x=0}^{x=a}\left(\frac{b}{a}\cdot\sqrt{a^2-x^2}-0\right)dx$$

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=b^2\left(\frac{a^2-x^2}{a^2}\right)$$

$$y=\frac{b}{a}\sqrt{a^2-x^2}$$

$$A=\frac{4b}{a}\int_0^a\sqrt{a^2-x^2}\;dx$$

$x=a\sin t$
$dx=a\cos t$

$x=0$ için
$a\sin t=0$
$\sin t=0$
$t=0$
$a\sin t=a$
$\sin t=1$
$t=\frac{\pi}{2}$

$$\frac{4b}{a}\int_0^{\pi/2}\sqrt{\underbrace{a^2-a^2\sin^2 t}_{a^2(1-\sin^2 t)}}\;a\cos t\,dt$$

$$\frac{4b}{a}\,a^2\int_0^{\pi/2}\cos^2 t\,dt = 4ba\int_0^{\pi/2}\frac{1+\cos 2t}{2}$$

$$=4ba\left(\frac{t}{2}+\frac{\sin 2t}{4}\right)\Bigg|_{t=0}^{t=\pi/2} = 4ba\left(\frac{\pi}{4}+\frac{\sin 2\cdot\frac{\pi}{2}}{4}-0\right)$$

$$=4ba\left(\frac{\pi}{4}+\underbrace{\sin\frac{\pi}{4}}_{0}\right)=\pi ab$$

Ör: $y = -x^2 - 2x + 3$ parabolü ve bu parabole $(-2, 3)$ nok teğet geçen teğet doğrusu ile y ekseni ~~ile~~ arasında kalan alan nedir?

$x = 0$ için $y = 3$ $(0, 3)$

$y = 0$ için $-x^2 - 2x + 3 = 0$

$x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$

$x = -3$ $B(-3, 0)$

$x = 1$ $C(1, 0)$

$T(x_0, y_0)$

$x_0 = \frac{-b}{2a} = \frac{2}{-2} = -1$

$-1 + 2 + 3 = 4$ $T(-1, 4)$

$a = -1 < 0$ (kollar aşağı)

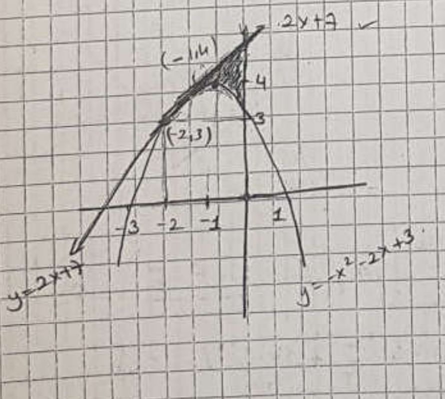
Teğet doğrusu $A(x_1, y_1)$ m eğimi belli olan teğet doğrusu,

$y - y_1 = m(x - x_1)$

$m = y' \Big|_{y = x_1}$ $m = -2x - 2 \Big|_{x=-2} \rightarrow -2(-2) - 2 = 2$

$A(-2, 3)$ $m = 2$ olan teğet doğrusu

$y - 3 = 2(x + 2)$



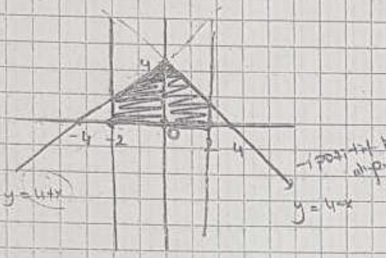
$$A = \int_{x=-2}^{x=0} (2x+7) - (-x^2 - 2x + 3)\, dx$$

$$\int_{-2}^{0} (x^2 + 4x + 4)\, dx \Rightarrow \int_{-2}^{0} (x+2)^2\, dx$$

$$\frac{(x+2)^3}{3} \Bigg|_{x=-2}^{x=0} \Rightarrow \frac{2^3}{3} - 0 = \frac{8}{3}$$

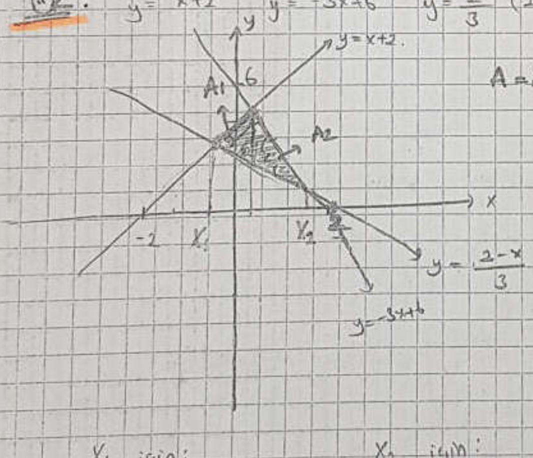
Ör: $y = 4 - |x|$ $x = 2$, $x = -2$, $y = 0$ aralarında kalan bölgenin alanı=?

$x < 0$ ise $4 + x$
$x > 0$ ise $4 - x$



$$A = 2\int_{x=0}^{x=2} (4 - x - 0)\,dx = 2\left(4x - \frac{x^2}{2}\right)\Bigg|_0^2 = 2(8 - 2)$$
$$= 12 \checkmark$$

Ör: $y = x + 2$ $y = -3x + 6$ $y = \frac{1}{3}(2 - x)$



$A = A_1 + A_2$

$$A = \int_{x_1=-1}^{x_2=1} \left(x + 2 - \left(\frac{2 - x}{3}\right)\right) dx + \int_{x_2=1}^{x=2} \left(-3x + 6 - \left(\frac{2 - x}{3}\right)\right) dx$$
$$= 4$$

X_1 için:

$\frac{2 - x}{3} = x + 2$

$2 - x = 3x + 6$

$4x = -4$

$x_1 = -1$

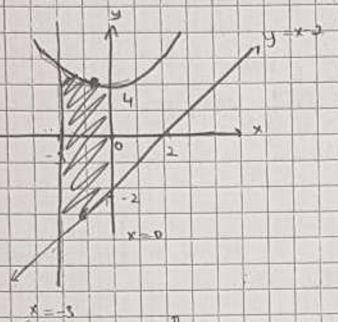
X_2 için:

$-3x + 6 = x + 2$

$4 = 4x$

$x_2 = 1$

Ör: $y=x^2+4$, $y=x-2$, $x=-3$, $x=0$



$x^2+4=x-2$

$x^2-x+6=0$

$\Delta=b^2-4ac = 1-4.1.6=-23<0$

kesişim yok.

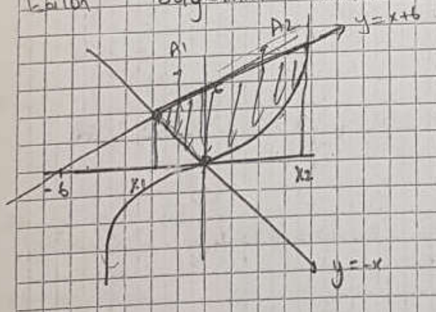
$$A=\int_{-3}^{0}\left(x^2+4-(x-2)\right)dx = \left(\frac{x^3}{3}-\frac{x^2}{2}+6x\right)\Bigg|_{x=-3}^{x=0}$$

$$=0-\left(\frac{-27}{3}-\frac{9}{2}-18\right) = \frac{27}{3}+\frac{9}{2}+18$$

$$=9+18+\frac{9}{2}$$

$$=27+\frac{9}{2}=\frac{54+9}{2}=\frac{63}{2}$$

Ör: $y=x^3$, $y=-x$, $y=x+6$ eğrileri ile doğru arasında kalan bölgenin alanını hesapla.



x_1 için:

$x+6=-x$

$2x=-6$

$x=-3$

x_2 için:

$x^3=x+6$

$x_2=2$ için

$2^3=2+6$

$8=8$ ✓

(sağladı)

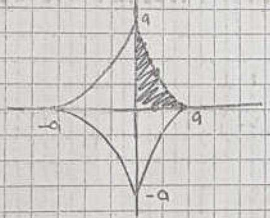
$$A = A_1 + A_2 \rightarrow \int_{x_1=-3}^{x_2=0} \left(x+6-(-x)\right)\,dx + \int_{x=0}^{x_2=2} \left(x+6-x^3\right)\,dx$$

$$= \int_{-3}^{0} (2x+6)\,dx + \int_{0}^{2} (x+6-x^3)\,dx = x^2+6x \Big|_{x=-3}^{x=0} + \left(\frac{x^2}{2} + 6x - \frac{x^4}{4}\right)\Big|_{0}^{2}$$

$$= 0 - (9-18) + (2+12-4)$$

$$= 9 + 10 = 19$$

Ö2: $x^{2/3} + y^{2/3} = a^{2/3}$ astroit eğrisinin alanını bulunuz.



$$a^{2/3}(\cos^3 t)^{2/3} + a^{2/3}(\sin^3 t)^{2/3}$$

$$= \underbrace{(\cos^2 t + \sin^2 t)}_{1} \cdot a^{2/3} = a^{2/3}$$

$$x = a \cdot \cos^3 t$$

$$y = a \cdot \sin^3 t$$

$$y^{2/3} = a^{2/3} - x^{2/3}$$

$$y = \left(a^{2/3} - x^{2/3}\right)^{\frac{3}{2}}$$

$$x = a \cdot \cos^3 t$$

$$dx = -3\cos^2 t \sin t\,dt$$

$$A = 4 \int_{x=0}^{x=a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx = 4 \int_{t=\pi/2}^{t=0} \left(a^{2/3} - \left(a\cos^3 t\right)^{2/3}\right)^{3/2} -3a\cos^2 t \sin t\,dt$$

$x = 0$ için,

$a\cos^3 t = 0$

$\cos t = 0$

$t = \pi/2$

$$= 12a \int_{0}^{\pi/2} \left(a^{2/3} - a^{2/3}\cos^2 t\right)^{3/2} \cdot \cos^2 t \sin t\,dt$$

$x = a$ için,

$a = a\cos^3 t$

$\cos^3 t = 1$

$\cos t = 1$

$t = 0$

$$= 12a \int_0^{\pi/2} \left(a^{2/3}\left(1-\cos^2 t\right)\right)^{3/2} \cos^2 t \sin t \, dt$$

$$= 12a^2 \int_0^{\pi/2} \left(\sin^2 t\right)^{3/2} \cos^2 t \sin t \, dt$$

$$= 12a^2 \int_0^{\pi/2} \cos^2 t \cdot \sin^4 t \, dt = 12a^2 \int_0^{\pi/2} \left(\frac{1+\cos 2t}{2}\right) \left(\frac{1-\cos 2t}{2}\right)^2 dt$$

$$\frac{\cancel{12}^{3} a^2}{\cancel{8}_{2}} \int_0^{\pi/2} (1+\cos 2t)(1-\cos 2t)^2 dt = \frac{3a^2}{2} \int_0^{\pi/2} (1+\cos 2t) \cdot (1-2\cos 2t+\cos^2 2t) \, dt$$

$$\frac{3a^2}{2} \int_0^{\pi/2} \left(1-2\cos 2t+\cos^2 2t+\cos 2t-2\cos^2 2t+\cos^3 2t\right) dt$$

$$\frac{3a^2}{2} \int_0^{\pi/2} \left(1-\cos 2t-\cos^2 2t+\cos^3 2t\right) dt = \frac{3a^2}{2} \int_0^{\pi/2} dt - \frac{3a^2}{2} \int_0^{\pi/2} \cos 2t \, dt$$

$$-\frac{3a^2}{2} \int_0^{\pi/2} \frac{1+\cos 4t}{2} dt + \frac{3a^2}{2} \int_0^{\pi/2} \cos^2 t \cos t \, dt$$

$$\frac{3a^2}{2} t \Big|_0^{\pi/2} - \frac{3a^2}{2} \left(\frac{\sin 2t}{2}\right) \Big|_0^{\pi/2} - \frac{3a^2}{2} \left(\frac{t}{2} - \frac{\sin 4t}{4 \cdot 2}\right) \Big|_0^{\pi/2}$$

$$+ \frac{3a^2}{2} \int_0^{\pi/2} \left(1-\sin^2 t\right) \cos t \, dt \quad \underline{\substack{\sin t = w \\ \cos t \, dt = dw}} \Rightarrow \frac{3a^2}{2} \int_0^{0} \left(1-w^2\right) \cdot dw$$

$$\frac{3a^2}{2} \cdot \frac{\pi}{2} - \frac{3a^2}{2} \left(\frac{\overset{0}{\sin \pi}}{2} - \sin 0\right) - \frac{3}{2} a^2 \left(\frac{\pi}{4} + \frac{\overset{0}{\sin 4 \cdot \frac{\pi}{2}}}{8} - 0\right)$$

$$+\frac{3a^2}{2}\int_{\alpha}^{\beta}(1-w^2)\,dw \Rightarrow \frac{3a^2\pi}{\underset{(2)}{4}} - \frac{3a^2\pi}{8} + \frac{3a^2}{2}\left(w-\frac{w^3}{3}\right)\Bigg|_{\alpha}^{\beta}$$

$$=\frac{3a^2\pi}{8}+\frac{3a^2}{2}\left(\sin t-\frac{\sin^3 t}{3}\right)\Bigg|_{0}^{\pi/2}$$

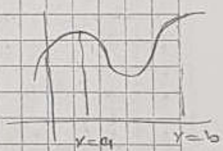
$$=\frac{3a^2\pi}{8}+\frac{3a^2}{2}\left(\sin\frac{\pi}{2}-\frac{\sin^3\frac{\pi}{2}}{3}-0\right) \Rightarrow \frac{3a^2\pi}{8}+a^2$$

* İntegral Hesabı Yöntemiyle Eğri Uzunluğu Hesabı (Yay uzunluğu hesabı) :

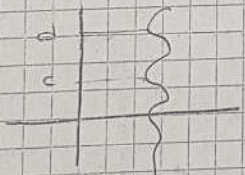
$y=f(x)$ şeklinde verilmiş olan bir eğrinin $[a,b]$ üzerindeki yay uzunluğunun veya eğri parçasının uzunluğu

Kartezyende

$$l=\int_{x=a}^{x=b}\sqrt{1+(y')^2}\,dx$$ biçiminde bulunur.



Benzer şekilde, $x=g(y)$ şeklinde verilmiş olan bir eğrinin $[c,d]$



$$l=\int_{c}^{d}\sqrt{1+(g'(y))^2}\,dy$$ şeklinde bulunur.

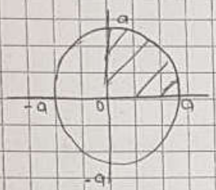
ÖR: a yarıçaplı çemberin çevre uzunluğunu hesaplayınız

$\sqrt{1+(y')^2}$

$x^2+y^2=a^2$

$x^2+y^2-a^2=0$

$2x+2y\cdot y'=0$



$$l = 4\int_{x=0}^{x=a} \sqrt{1+(y')^2}\,dx$$

$$4\int_a^0 \sqrt{1+\left(\frac{-x}{y}\right)^2}\,dx \Rightarrow 4\int_a^0 \sqrt{1+\frac{x^2}{y^2}}\,dx$$

$x^2+y^2-a^2=0$

$2x+2y\cdot y'=0$

$y'=\frac{-x}{y}$ ✓

$$=4\int_a^0 \sqrt{1+\frac{x^2}{y^2}}\,dx \Rightarrow 4\int_a^0 \sqrt{\frac{x^2+y^2}{y^2}}\,dx$$

$$=4\int_0^a \sqrt{\frac{a^2}{y^2}}\,dx = 4\int_a^0 \frac{a}{y}\,dx = 4a\int_a^a \frac{dx}{\sqrt{a^2-x^2}}$$

$$=4a\left(\arcsin\left(\frac{x}{a}\right)\Big|_0^a\right) \Rightarrow 4a\Big(\underbrace{\arcsin 1}_{\pi/2}-\underbrace{\arcsin 0}_{0}\Big)=4a\frac{\pi}{2}$$

$=2a\pi$.

ÖR: $y=\ln x-\frac{x^2}{8}$ eğrisinin $x=2$ ve $x=4$ apsisli nokarasında kalan parçasının uzunluğunu bul.

$y'=\frac{1}{x}-\frac{x}{4}$

$1+(y')^2=1+\left(\frac{1}{x}-\frac{x}{4}\right)^2$

$=1+\left(\frac{1}{x^2}-\frac{1}{2}+\frac{x^2}{16}\right)$

$=\frac{1}{x^2}+\frac{1}{2}+\frac{x^2}{16}=\left(\frac{1}{x}+\frac{x}{4}\right)^2$

$\left(\frac{1}{x}\right)^2 \qquad \left(\frac{x}{4}\right)^2$

$\frac{1}{\cancel{x}}\cdot\frac{\cancel{x}}{4}\cdot 2=\frac{1}{2}$

$$l = \int_{x=2}^{x=4} \sqrt{1+(y')^2}\, dx = \int_2^4 \sqrt{\left(\frac{1}{x}+\frac{x}{4}\right)^2} = \int_2^4 \left(\frac{1}{x}+\frac{x}{4}\right) dx$$

$$= \left(\ln x + \frac{x^2}{8}\right)\Bigg|_{x=2}^{x=4} \Rightarrow \ln 4 + \frac{4^2}{8} - \ln 2 - \frac{2^2}{8}$$

$$= \ln 4 - \ln 2 + \frac{16}{8} - \frac{4}{8}$$

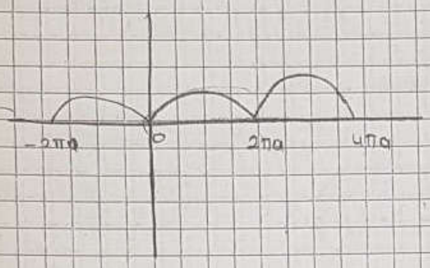
$$\Rightarrow \ln\frac{4}{2} + \frac{12}{8} \Rightarrow \ln 2 + \frac{3}{2}$$

NOT: $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $t_1 \le t \le t_2$ parametrik denklemiyle verilmiş eğrinin $[t_1, t_2]$ aralığındaki parçasının uzunluğu

$$l = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$ şeklinde hesaplanır.

$$L = \int_0^{2\pi} \sqrt{(a - a.\sin\theta)^2 + (a.\cos\theta)^2}\, d\theta \rightarrow a \int_0^{2\pi} d\theta = a.\theta \Big|_0^{2\pi} = 2\pi a$$

Ör: Parametrik denklemi: $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$, $0 \le t \le 2\pi$ olan sikloit eğrisinin uzunluğunu bulunuz.



$x'(t) = a(1-\cos t)$

$y'(t) = a\sin t$

$$(x'(t))^2 + (y'(t))^2 = a^2(1 - 2\cos t + \cos^2 t + \underbrace{\cos^2 t + \sin^2 t}_{1})$$

$$= a^2(2 - 2\cos t)$$

$$= 2a^2(1-\cos t)$$

$$(x'(t))^2 + (y'(t))^2 = 2a^2(1-\cos t) \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$= 2a^2\left(1 - \left(\cos^2\frac{t}{2} - \sin^2\frac{t}{2}\right)\right) \qquad 1-\cos^2\frac{t}{2} = \sin^2\frac{t}{2}$$

$$= 2a^2\left(\sin^2\frac{t}{2} + \sin^2\frac{t}{2}\right) = 4a^2\sin^2\frac{t}{2}$$

$$l = \int_{2\pi}^{0}\sqrt{4a^2 \cdot \sin^2\frac{t}{2}}\,dt \Rightarrow 2a\int_{0}^{2\pi}\sin\frac{t}{2}\,dt$$

$$= 2a\left(\frac{-\cos\frac{t}{2}}{\frac{1}{2}}\right)\Bigg|_{0}^{2\pi} \Rightarrow -4a\left(\cos\frac{t}{2}\right)\Bigg|_{0}^{2\pi} = -4a(\cos\pi - \cos 0)$$

$$= -4a(-1-1)$$

$$= 8a$$

Ör: $x = \frac{y^4}{4} + \frac{1}{8y^2}$ eğrisinin $1 \le y \le 2$ parçası arasında kalan eğrinin uzunluğunu bulunuz.

$$l = \int_{y=1}^{y=2} \sqrt{1 + \left[\left(\frac{y^4}{4} + \frac{1}{8y^2}\right)'\right]^2}\, dy = \int_1^2 \sqrt{1 + \left(\frac{4y^3}{4} - \frac{1}{4y^3}\right)^2}\, dy$$

$$\int_1^2 \sqrt{1 + y^6 - \frac{1}{2} + \frac{1}{16y^6}}\, dy = \int_1^2 \sqrt{y^6 + \frac{1}{2} + \frac{1}{16y^6}}\, dy$$

$$(y^3)^2 \qquad \left(\frac{1}{4y^3}\right)^2$$

$$y^3 \quad \frac{1}{4y^3} \quad 2$$

$$= \int_1^2 \sqrt{\left(y^3 + \frac{1}{4y^3}\right)^2}\, dy = \int_1^2 \left(y^3 + \frac{1}{4y^3}\right) dy$$

$$\frac{y^4}{4} + \frac{1}{4}\frac{y^{-2}}{-2}\Bigg|_1^2 = \frac{y^4}{4} - \frac{1}{8y^2}\Bigg|_{y=1}^{y=2}$$

$$l = \int_1^2 \sqrt{1 + [x'(y)]^2}\, dy \qquad x'_y = y^3 - \frac{1}{8y^4}$$

UYGULAMALAR

1) $\int_0^2 x \cdot |1-x^2| \, dx = ?$

$1-x^2=0 \quad , \quad x_{1,2} = \mp 1$

x		-1		$+1$	
$1-x^2$	$-$	0	$+$	0	$-$

$x \in [0,1] \quad 1-x^2 > 0 \rightarrow 1-x^2$

$x \in [1,2] \quad -(1-x^2) < 0 \rightarrow -(1-x^2)$

$$I = \int_0^1 x(1-x^2)\,dx + \int_1^2 -x(1-x^2)\,dx$$

$$\int_0^1 (x-x^3)\,dx + \int_1^2 (-x+x^3)\,dx$$

$$\left.\frac{x^2}{2} - \frac{x^4}{4}\right|_0^1 + \left.\frac{-x^2}{2} + \frac{x^4}{4}\right|_1^2$$

$$\frac{1}{2} - \frac{1}{4} - 0 - \left(\frac{2^2}{2} - \frac{2^4}{4} - \left(\frac{1}{2} - \frac{1}{4}\right)\right)$$

$$= \frac{1}{4} - \left(2 - 4 - \frac{1}{2} + \frac{1}{4}\right)$$

$$= \cancel{\frac{1}{4}} - 2 + 4 + \frac{1}{2} - \cancel{\frac{1}{4}}$$

$$= 2 + \frac{1}{2} = \frac{5}{2}$$

e/ $\displaystyle\int_0^4 \operatorname{sgn}(x^2-4x+3)\,dx = ?$

$f(x) = x^2-4x+3 \rightarrow (x-1)(x-3)=0$

$x_1 = 1 \quad x_2 = 3$

x		1		3	
$f(x)$	+	0	−	0	+
$\operatorname{sgn}(f(x))$	+1		−1		+1

0 — 1 — 3 — 4

$$I = \int_0^1 \operatorname{sgn}(x^2-4x+3)\,dx + \int_1^3 \operatorname{sgn}(x^2-4x+3)\,dx + \int_3^4 \operatorname{sgn}(x^2-4x+3)\,dx$$

$$= \int_0^1 1\,dx + \int_1^3 -1\,dx + \int_3^4 1\,dx$$

$$x\Big|_0^1 - x\Big|_1^3 + x\Big|_3^4$$

$$1-(3-1)+4-3$$

$$1-2+1 = 0$$

Ör/ $\displaystyle\int_0^{5\pi/4} \frac{\sin 2x\,dx}{\sin^4 x+\cos^4 x} = ?$

$\sin^2 x = u$

$\sin 2x\,dx = du$

$\cos^4 x = (\cos^2 x)^2 = (1-\sin^2 x)^2$

$= (1-u)^2$

$I = \displaystyle\int_\alpha^\beta \frac{du}{u^2+(1-u)^2} = \int_\alpha^\beta \frac{du}{u^2+1-2u+u^2}$

$= \displaystyle\int_\alpha^\beta \frac{du}{2u^2-2u+1} = \frac{1}{2}\int_\alpha^\beta \frac{du}{u^2-u+\frac{1}{2}}$

tam kare yapmak için

$u^2-u \Rightarrow \left(u-\frac{1}{2}\right)^2-\frac{1}{4}$

$\displaystyle\frac{1}{2}\int_\alpha^\beta \frac{du}{\left(u-\frac{1}{2}\right)^2-\underset{(1)}{\frac{1}{4}}+\underset{(2)}{\frac{1}{2}}} \Rightarrow \frac{1}{2}\int_\alpha^\beta \frac{du}{\left(u-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$

$\displaystyle\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{u-\frac{1}{2}}{\frac{1}{2}}\right)\Bigg|_\alpha^\beta$

$= \arctan(2u-1)\Big|_\alpha^\beta \rightarrow \arctan\left(2\sin^2 x-1\right)\Big|_0^{5\pi/4}$

$= \arctan\left(2\cdot\sin^2\frac{5\pi}{4}-1\right) - \arctan\left(2\sin^2 0-1\right)$

$= -\arctan(-1)$ → tek fonksiyon old. için

$= \arctan 1 \rightarrow \pi/4$

Q/ $\displaystyle\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$

$2x+1 = u^2$

$\cancel{2}\,dx = \cancel{2}u\,du$

$dx = u\,du$

$\displaystyle I = \int_\alpha^\beta \frac{u+1-1}{1+u}$

$\displaystyle = \int_\alpha^\beta \frac{u+\cancel{1}}{u+1}\,du - \int_\alpha^\beta \frac{1}{u+1}\,du$

$\displaystyle = \int_\alpha^\beta du - \int_\alpha^\beta \frac{1}{u+1}\,du \rightarrow u\Big|_\alpha^\beta - \ln|u+1|\Big|_\alpha^\beta$

$\displaystyle = \sqrt{2x+1}\,\Big|_0^4 - \ln\left|\sqrt{2x+1}+1\right|\Big|_0^4$

$= 3-1-(\ln|4| - \ln|2|)$

$= 2 - \ln 2$

Q/ $\displaystyle\int_0^1 \frac{x}{x^2+3x+2}\,dx = ?$

1) $\displaystyle \frac{x}{(x+2)(x+1)} = \frac{A}{\underset{(x+2)}{x+1}} + \frac{B}{\underset{(x+1)}{x+2}}$

(1)

$x = A(x+2) + B(x+1)$

$x = x(\underbrace{A+B}_{1}) + \underbrace{2A+B}_{0}$

$A = -1$

$B = 2$

2) $\displaystyle I = \int_1^0 \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$

$= \left(-\ln|x+1| + 2\ln|x+2|\right)\Big|_0^1$

$= 2\ln|x+2| - \ln|x+1|$

$\displaystyle \ln\left|\frac{(x+2)^2}{x+1}\right|\Big|_0^1$

$\displaystyle \ln\frac{9}{2} - \ln 4$

$\displaystyle = \ln\left(\frac{\frac{9}{2}}{4}\right) = \ln\left(\frac{9}{8}\right)$

kısmi integrasyon için çarpım şeklinde olmalı.

(11) $$\int_{\pi}^{2\pi} \frac{\tan^2 x}{x-\tan x}\,dx = ?$$

$x - \tan x = u$

$1 - (1+\tan^2 x)\,dx = du$

$-\tan^2 x\,dx = du$

$$\int_{\alpha}^{\beta} \frac{du}{u} = -\ln u \Big|_{\alpha}^{\beta} \rightarrow -\ln|x-\tan x| \Big|_{\pi}^{2\pi}$$

$$= -\left(\ln|2\pi - \tan 2\pi| - \ln|\pi - \tan\pi|\right)$$

$$= -\left(\ln 2\pi - \ln\pi\right)$$

$$= -\ln 2$$

(12) $$\int_{0}^{1} \frac{x^2}{\sqrt{4-x^2}}\,dx$$

$\sqrt{2^2 - x^2}$; $x = 2\sin t$ dön. yapılır.

$dx = 2\cos t\,dt$

$x=0$ için $0 = 2\sin t$
$t = 0$

$x=1$ için $1 = 2\sin t$
$\frac{1}{2} = \sin t$
$t = \frac{\pi}{6}$

$$I = \int_{t_1=0}^{t_2=\pi/6} \frac{4\sin^2 t}{\sqrt{4-4\sin^2 t}} \cdot 2\cos t\,dt$$

$$= \int_0^{\pi/6} 4\sin^2 t\,dt$$

$$= \cancel{4}^{2} \int_0^{\pi/6} \frac{1-\cos 2t}{\cancel{2}}$$

$$= 2\left(t - \frac{\sin 2t}{2}\right)\Big|_0^{\pi/6}$$

$$= 2\cdot\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

(11)R/ $\displaystyle\int_0^{\pi} \frac{dx}{3+2\cos x} = ?$

$\tan\frac{x}{2} = u$

$2\arctan u = x$

$\frac{2}{1+u^2}du$

$\cos x = \frac{1-u^2}{1+u^2}$

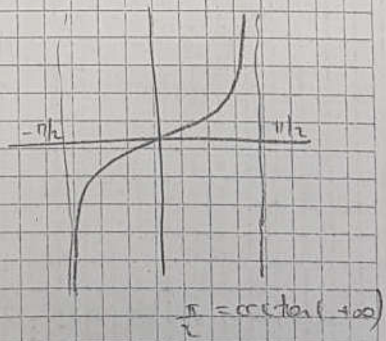
$$I = \int_\alpha^\beta \frac{\frac{2du}{1+u^2}}{3+2\cdot\left(\frac{1-u^2}{1+u^2}\right)} = \int_\alpha^\beta \frac{\frac{2du}{1+u^2}}{\frac{3+3u^2+2-2u^2}{1+u^2}} \to \int_\alpha^\beta \frac{2\,du}{u^2+(\sqrt5)^2}$$

$$2\int_\alpha^\beta \frac{du}{u^2+(\sqrt5)^2} \to 2\arctan\left(\frac{u}{\sqrt5}\right)\Bigg|_\alpha^\beta \to 2\arctan\left(\frac{\tan\frac{x}{2}}{\sqrt5}\right)\Bigg|_0^\pi$$

$$\frac{2}{\sqrt5}\left(\arctan\left(\frac{\tan\frac{\pi}{2}}{\sqrt5}\right) - \underbrace{\arctan\left(\frac{\tan\frac{0}{2}}{\sqrt5}\right)}_{0}\right)$$

$$= \frac{2}{\sqrt5}\arctan(\infty)$$

$$\to \frac{\cancel{2}}{\sqrt5}\cdot\frac{\pi}{\cancel{2}} = \frac{\pi}{\sqrt5}$$



$\frac{\pi}{2} = \arctan(+\infty)$

Q2) $\displaystyle\int_0^{\ln 5} \frac{e^x \cdot \sqrt{e^x - 1}}{e^x + 3}\, dx = ?$

$e^x - 1 = u^2$

$e^x\, dx = 2u\, du$

$$\int_\alpha^\beta \frac{\sqrt{u^2} \cdot 2u\,du}{u^2+4} = \int_\alpha^\beta \frac{2u^2}{u^2+4}\, du$$

$$= 2\int_\alpha^\beta \frac{u^2+4-4}{u^2+4}\, du \rightarrow 2\left(\int_\alpha^\beta \frac{\cancel{u^2+4}}{\cancel{u^2+4}}\, du - 4\int_\alpha^\beta \frac{du}{4+u^2}\right)$$

$$2u \Big|_\alpha^\beta - 8\frac{1}{2} \arctan \frac{u}{2} \Big|_\alpha^\beta$$

$$2 \cdot \sqrt{e^x - 1}\Big|_0^{\ln 5} - 4 \arctan\left(\frac{\sqrt{e^x-1}}{2}\right)\Big|_0^{x=\ln 5}$$

$$2 \cdot \sqrt{e^{\ln 5} - 1} - \sqrt{e^0 - 1}\,)$$

$$-4\left(\arctan\left(\frac{\sqrt{e^{\ln 5} - 1}}{2}\right)\right) - \arctan\left(\frac{\sqrt{5}-1}{2}\right) \quad \underbrace{-\arctan 0}_{0}\;)$$

$= 4 - 4 \arctan 1$

$= 4 - 4 \cdot \frac{\pi}{4}$

$= 4 - \pi$

ÖR/ $$\int_{-\pi}^{\pi} |\cos^3 x| \, dx = ?$$

Kural: $$\int_{-\alpha}^{\alpha} f(x)\,dx = \begin{cases} 0, & f(x) \text{ tek ise} \\ 2\int_{0}^{\alpha} f(x)\,dx, & f(x) \text{ çift ise} \end{cases}$$

$$f(x) = |\cos^3 x|$$

$$f(-x) = |\cos^3(-x)| = |\cos^3 x| = f(x)$$

$$f(-x) = f(x)$$

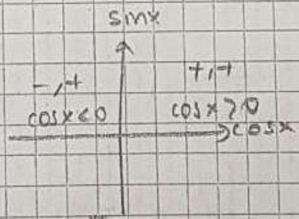
$f(x)$ çift

$$I = 2\int_{0}^{\pi} |\cos^3 x| \, dx$$

$$f(-x) = f(x)$$

$f(x)$ çift

$$I = 2\int_{0}^{\pi} |\cos^3 x| \, dx$$



$$I = 2\left(\int_{0}^{\pi/2} \underbrace{|\cos^3 x|}_{>0} \, dx + \int_{\pi/2}^{\pi} \underbrace{|\cos^3 x|}_{<0} \, dx \right)$$

$$= 2\left(\int_{0}^{\pi/2} \cos^{\overset{\text{tek}}{3}} x \, dx - \int_{\pi/2}^{\pi} \cos^3 x \, dx \right)$$

$$= 2\left(\int_{0}^{\pi/2} \cos^2 x \cos x \, dx - \int_{\pi/2}^{\pi} \cos^2 x \cos x \, dx \right)$$

$$= 2\left(\int_0^{\pi/2}(1-\sin^2 x)\,|\cos x|\,dx - \int_{\pi/2}^{\pi}(1-\sin^2 x)\cos x\,dx\right.$$

$\left[\begin{array}{l}\sin x = w \\ \cos x\,dx = dw\end{array}\right]$

$$= 2\left(\int_\alpha^\beta (1-w^2)\,dw - \int_{\alpha_1}^{\beta_1}(1-w^2)\,dw\right.$$

$$= 2\left(w-\frac{w^3}{3}\right)\Bigg|_\alpha^\beta - \left(w-\frac{w^3}{3}\right)\Bigg|_{\alpha_1}^{\beta_1}$$

$$= 2\left(\left(\sin x-\frac{\sin^3 x}{3}\right)\Bigg|_{x=0}^{x=\pi/2} - \left(\sin x-\frac{\sin^3 x}{3}\right)\Bigg|_{x=\pi/2}^{x=\pi}\right)$$

$$= 2\left(1-\frac{1}{3}-\left(\sin\pi-\frac{\sin^3\pi}{3}-\left(1-\frac{1}{3}\right)\right)\right)$$

$$= 2\left(\frac{2}{3}+\frac{2}{3}\right) = \frac{8}{3}$$

11) $$\frac{1}{6} \leq \int_0^1 \frac{x^2\,dx}{1+x^{10}} \leq \frac{1}{3}$$

$$x \in [0,1] \qquad (x^2 > 0)$$

$$0 \leq x \leq 1$$

$$0 \leq x^{10} \leq 1$$

$$1 \leq 1+x^{10} \leq 2$$

$$\frac{1}{2} \leq \frac{1}{1+x^{10}} \leq 1$$

$$\frac{x^2}{2} \leq \frac{x^2}{1+x^{10}} \leq x^2$$

$$\int_0^1 \frac{x^2}{2}\,dx \leq \underbrace{\int_0^1 \frac{x^2}{1+x^{10}}}_{A} \leq \int_0^1 x^2\,dx$$

$$\left.\frac{x^3}{6}\right|_0^1 \leq A \leq \left.\frac{x^3}{3}\right|_0^1$$

$$\frac{1}{6} \leq A \leq \frac{1}{3}$$

11) $$F(x) = \int_{x^2}^{x^3} \sqrt{1+t^2}\,dt \quad \text{ise} \quad F'(x) = ?$$

Leibnitz Kuralı:

$$F(x) = \int_{f(x)}^{g(x)} \varphi(t)\,dt \qquad F'(x) = \varphi(g(x)) \cdot g'(x) - \varphi(f(x)) \cdot f'(x).$$

$$F'(x) = \sqrt{1+(x^3)^2}\cdot \underbrace{3x^2}_{(x^3)'} - \sqrt{1+(x^2)^2}\cdot \underbrace{2x}_{(x^2)'}$$

Ör/ Sürekli f fonk için $f(2)=3$ ise

$$\lim_{x\to 2} \frac{x}{x-2} \int_2^x f(t)\,dt = ?$$

$$F(x) = \int_2^x f(t)\,dt$$ denilirse.

$$F'(t) \overset{\text{leibnitz}}{=\!=} f(x)\cdot x' - \underbrace{f(2)\cdot 2'}_{0}$$

$$F'(x) = f(x)$$

$$F'(2) = f(2) = 3$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a) \text{ dır.}$$

$$F'(2) = \lim_{x\to 2} \frac{F(x)-F(2)}{x-2} = \lim_{x\to 2} \frac{\int_2^x f(t)\,dt - \overbrace{\int_2^2 f(t)\,dt}^{0}}{x-2}$$

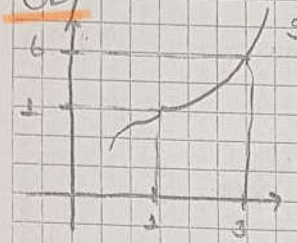
$$= \lim_{x\to 2} \frac{1}{x-2}\int_2^x f(t)\,dt$$

$$\lim_{x\to 2} \frac{x}{x-2} \int_2^x f(t)\,dt = \lim_{x\to 2} x \cdot \lim_{x\to 2} \frac{1}{x-2}\int_2^x f(t)\,dt$$

$$= 2\cdot F'(2) \Rightarrow 2\cdot 3 = 6$$

?

11)



Sürekli türeve sahip bir f fonksiyonu için yukarıdaki şekil dikkate alınarak,

$$\int_1^3 \frac{f'(x)}{x}\,dx - \int_1^3 \frac{f(x)}{x^2}\,dx = ?$$

$$I = \int_1^3 \left(\frac{f'(x)}{\underset{(x)}{x}} - \frac{f(x)}{x^2} \right) dx = \int_1^3 \underbrace{\frac{f'(x)\cdot x - f(x)}{x^2}}_{\left(\frac{f(x)}{x}\right)'} dx$$

$$= \int_1^3 \left(\frac{f(x)}{x} \right)' dx = \frac{f(x)}{x} \Bigg|_{x=1}^{x=3} = \frac{f(3)}{3} - \frac{f(1)}{1} = \frac{6}{3} - \frac{1}{1}$$

$$\Rightarrow 2 - 1 = 1 \checkmark$$

12) Sürekli bir f fonk. için,

$$\int_0^{\pi/2} f(\sin x)\,dx = \int_0^{\pi/2} f(\cos x)\,dx$$ old. göster.

$$\left(x = \frac{\pi}{2} - t \right.$$

$$\left. dx = -dt \right)$$

$$\int_0^{\pi/2} f\left(\sin\left(\frac{\pi}{2} - t\right)\right) dt$$

$$\left[\; x = \frac{\pi}{2} - t \right.$$

$$dx = -dt$$

$x = 0$ için $0 = \frac{\pi}{2} - t$

$t = \pi/2 \checkmark$

$x = \pi/2$ için $\pi/2 = \pi/2 - t$

$$\left. t = 0 \;\right]$$

$$I = \int_0^{\pi/2} (-1)\, f\left(\sin\left(\frac{\pi}{2} - t\right)\right) dt$$

$$\int_\alpha^\beta f(x)\,dx = \int_\alpha^\beta f(t)\,dt$$

$$\int_0^\pi \sin x\,dx = \int_0^\pi \sin t\,dt$$

$$= \int_0^{\pi/2} f(\cos x)\,dt = \int_0^{\pi/2} f(\cos x)\,dx$$

4/

$$\int_{-1}^{+1} \underbrace{\frac{x^5 + x^3 - 2x}{\cos^2 x}}_{\text{Tek midir çift midir bak!}} dx = ?$$

$f(-x) = f(x) \Rightarrow f$ çift

$f(-x) = -f(x) \Rightarrow f$ tek

-1'den +1'e old. için simetrik aralık

$$f(-x) = \frac{(-x)^5 + (-x)^3 - 2(-x)}{\underbrace{\cos^2(-x)}_{\cos^2 x}} \Rightarrow \frac{-x^5 - x^3 + 2x}{\cos^2 x}$$

$$= -\underbrace{\left(\frac{x^5 + x^3 - 2x}{\cos^2 x}\right)}_{f(x)} = -f(x)$$

f tek

f tek olduğu için $\int_{-a}^{+a} f(x)\,dx = 0$

$= 0$ ✓

42/ $y^2 = 2x - 5$ eğrisi ile $y = x - 4$ doğrusu arasında kalan bölgenin alanını hesaplayınız.

$x = \frac{y^2+5}{2}$

$y = 0$ için $x = \frac{5}{2}$. $\left(\frac{5}{2}, 0\right)$

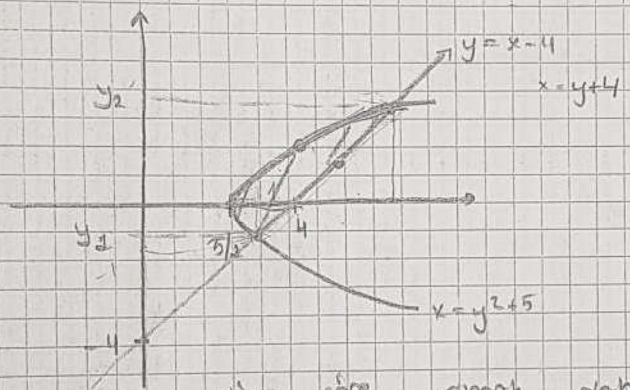
$x = 0$ için $y^2 = -5$ y yok.

$y = x - 4$

$x = 0$ için $y = -4$

$y = 0$ için $x = 4$

$(4, -4)$



y'ye göre yazmak daha kolay

$\frac{y^2+5}{2} = y+4$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y_1 = -1$

$y_2 = 3$

$\text{Alan} = \int_{y_1}^{y_2} (f\,\text{sağ} - f\,\text{sol})\, dy$

x'i y cinsinden bul

$A = \int_{-1}^{3} \left(y+4 - \frac{y^2+5}{2}\right) dy$

$\left(\frac{y^2}{2} + 4y - \frac{y^3}{6} - \frac{5}{2}y\right)\Bigg|_{y=-1}^{y=3}$

$= \frac{9}{2}$

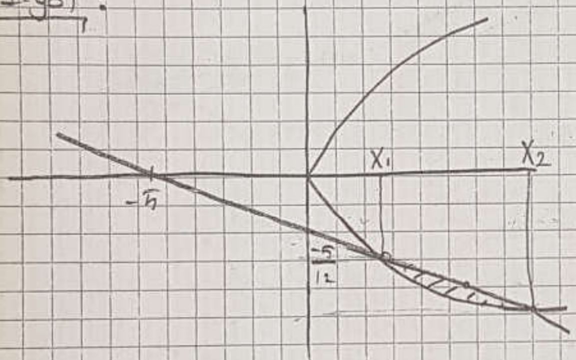
4/ b) $x = 4y^2$ eğrisi ile $x + 12y + 5 = 0$ doğrusu arasında kalan bölgenin alanı?

$y = 0 \quad x = 0$ (orjin) $A(0,0)$

$x = 0 \quad y = -\frac{5}{12}$

$y = 0$ için $x = -5$

1. yol :



$y^2 = \frac{x}{4} \qquad y = \mp \frac{\sqrt{x}}{2}$

4. bölge old. için (-) 'lisini alıyoruz.

$y = -\frac{\sqrt{x}}{2}$

$y = \frac{-x-5}{12}$

$\left(-\frac{\sqrt{x}}{2}\right)^2 = \left(\frac{-x-5}{12}\right)^2$

$\frac{x}{4} = \frac{x^2+10x+25}{\cancel{144}\ 36}$

$x^2 + 10x + 25 - 36x = 0$

$x^2 - 26x + 25 = 0$

$(x-1)(x-25) = 0$

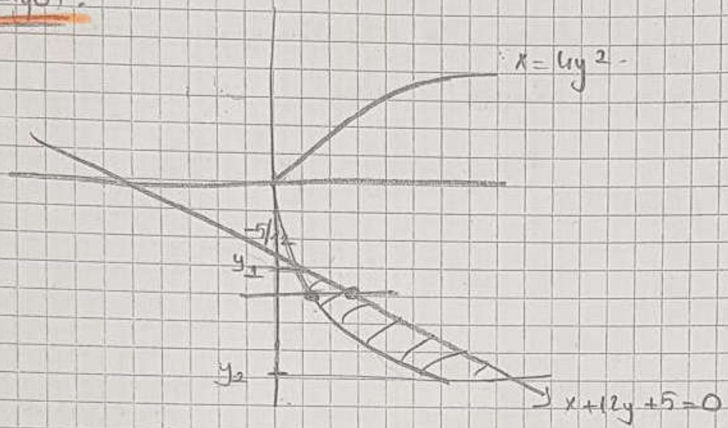
$x_1 = 1 \quad x_2 = 25$

x'e göre alan hesabı :

$A = \int_{x_1}^{x_2} \left(f(x) - g(x) \right) dx$

$= \int_{1}^{25} \left(\frac{-x-5}{12} + \frac{\sqrt{x}}{2} \right) dx = \frac{16}{3}$ ✓

2. yol:



$$A=\int_{y_1}^{y_2}(f_{sağ}-f_{sol})\,dy$$

x'in y tününden değerini al

$$4y^2=-12y-5$$

$$4y^2+12y+5=0$$

$$(2y+1)(2y+5)=0$$

$$y_1=\frac{-1}{2}\qquad y_2=\frac{-5}{2}$$

$$A=\int_{y_1=\frac{-5}{2}}^{y_2=\frac{-1}{2}}\left(-12y-5-4y^2\right)dy=\frac{16}{3}$$

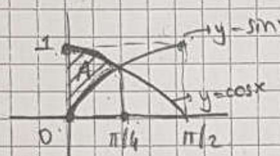
(1) Ör/ 1. bölgede $y = \sin x$, $y = \cos x$ eğrileri ile $x = 0$ doğrusu arasında kalan bölgenin alanını bulunuz

$\sin 0 = 0$ $\quad$ $\cos 0 = 1$

$\sin \pi/2 = 1$ $\quad$ $\cos \pi/2 = 0$

$\sin x = \cos x$

$x = \pi/4$



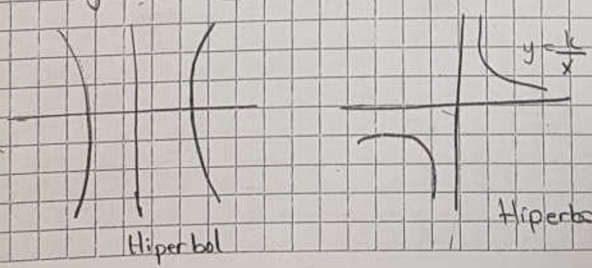
$$A = \int_{x=0}^{x=\pi/4} (\cos x - \sin x)\,dx \rightarrow \sin x + \cos x \Big|_{0=x}^{\pi/4=x}$$

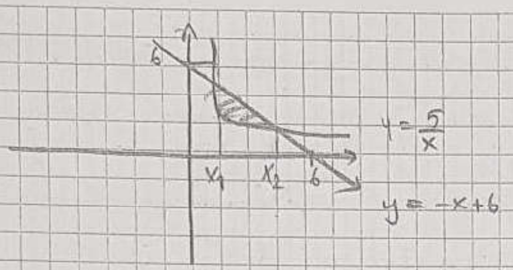
$$\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 0 + \cos 0)$$

$$= \frac{1}{\sqrt{2}} \cdot 2 - 1 = \frac{2}{\sqrt{2}} - 1$$

(2) Ör/ $x \cdot y = 5$ hiperbol eğrisi ile $y = -x + 6$ doğru denk. arasında kalan bölgenin alanı=?

$y^2 - x^2 = k^2$





$$\frac{5}{x} = -x+6$$

$$5 = -x^2+6x$$

$$x^2-6x+5=0 \qquad (x-1)(x-5)=0$$

$$x_1=1$$

$$x_2=5$$

$$A=\int_1^5\left((-x+6)-\frac{5}{x}\right)dx$$

$$=\frac{-x^2}{2}+6x-5\ln x$$

14) $x=\ln(\sec y)$ denklemi ve $y=0$, $y=\pi/3$ uç noktalarının ordinatları verilmiş eğri parçasının uzunluğunu bulunuz.

$$x=\underbrace{\ln(\sec y)}_{g(y)}$$

$$g'(y)=\frac{(\sec y)'}{\sec y}=\frac{\left(\frac{1}{\cos y}\right)'}{\frac{1}{\cos y}}$$

$$=\frac{\frac{\sin y}{\cos^2 y}}{\frac{1}{\cos y}}=\frac{\sin y}{\cos y}=\tan y$$

$$\sqrt{1+(g'(y))^2}=\sqrt{1+\tan^2 y}$$

$$=\sqrt{\frac{1}{\cos^2 y}}=\frac{1}{\cos y}$$

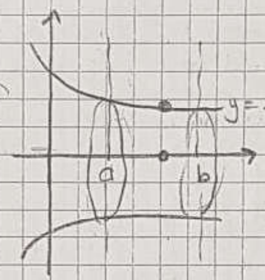
$$L=\int_0^{\pi/3}\frac{1}{\cos y}dy$$

$$=\ln\left|\frac{1+\sin y}{\cos y}\right|\Bigg|_0^{\pi/3}$$

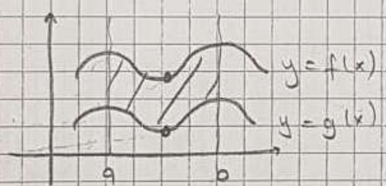
Hacim Hesabı:

$y = f(x)$ eğrisinin $x = a$, $x = b$ doğruları ile x ekseni arasında kalan bölgenin x ekse etrafında döndürülmesiyle meydana gelen dönel cismin hacmi,

$$V = \pi \int_a^b f^2(x)\,dx$$



NOT

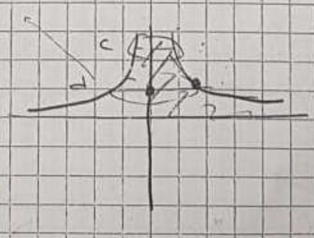


$$V = \pi \int_a^b \left(f^2(x) - g^2(x)\right) dx$$

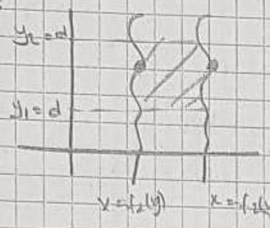
$x = g(y)$ olsa, $y = c$, $y = d$

$$V = \int_c^d g^2(y)\,dy$$

f^2(sağ) - f^2(sol) → hacim



NOT:



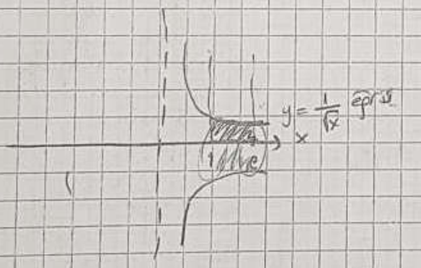
$$V = \pi \int_c^d \left(f_2^{\,2}(y) - f_1^{\,2}(y) \right) dy$$

Ör/ $y = \dfrac{1}{\sqrt{x}}$ eğrisi $x = 1$ ve $x = e$ doğruları ile x eks. arasında kalan bölgenin x ekseni etrafında dönmesiyle meydana gelen bölgenin hacmi?

$x = 0$ düşey asimptot

$x \to +\infty$, $y = 0$

$x \to 0$, $y = +\infty$



$$V = \pi \int_1^e \left(\left(\frac{1}{\sqrt{x}}\right)^2 - 0^2 \right) dx$$

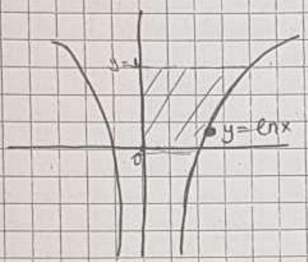
$$= \pi \int_1^e \frac{1}{x}\, dx \Rightarrow \pi \cdot (\ln x) \Big|_1^e$$

$$= \pi (\ln e - \ln 1)$$

$$= \pi.$$

Ör/ $y = \ln x$ eğrisi x eks. y eks $y = 1$ ekseni arasında kalan y eks. etrafında dön.

$$\xrightarrow{x = e^y}$$



$$V = \pi \int_{0}^{1} \left((e^y)^2 - 0^2 \right) dy$$

$$= \pi \int_{y=0}^{y=1} e^{2y}\, dy \rightarrow \pi \frac{e^{2y}}{2} \Big|_0^1$$

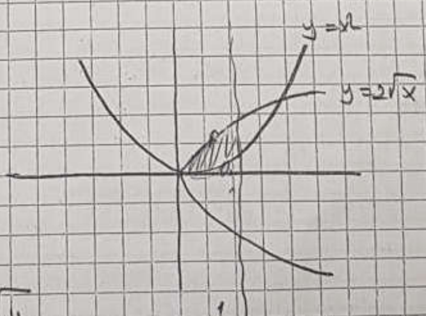
$$= \frac{\pi}{2} e^{2y} \Big|_0^1$$

$$= \frac{\pi}{2} (e^2 - 1) \text{ br}^3$$

Ör/ $y = 2\sqrt{x}$, $y = x^2$ eğrileri ile $x = 1$ doğrusu tarafından sınırlanan bölgenin x ekseni etrafında döndürülmesiyle meydana gelen dönel cismin hacmini bulun.

$y = 2\sqrt{x}$ $\quad$ $y = 2\sqrt{x} > 0$

$x = \frac{y^2}{4} > 0$



$2\sqrt{x} = x^2$

$4x = x^4$

$x^4 - 4x = 0$

$x(x^3 - 4) = 0$

$x_1 = 0 \quad x_2 = \sqrt[3]{4}$

$$V = \pi \int_0^1 \left((2\sqrt{x})^2 - (x^2)^2 \right) dx$$

$$\pi \cdot \left(\frac{4x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1 \Rightarrow \pi \left(2 - \frac{1}{5} \right) = \frac{9\pi}{5} \text{ br}^3$$

Q) $0<a<b$ olsun. Merkezi $(0,b)$'de bulunan a yarıçaplı bir çember tarafından sınırlanan bölgenin x eksi etrafında döndürülmesiyle meydana gelen dönel cismin hacmini hesaplayınız.

$M(0,b)$ $r=a$

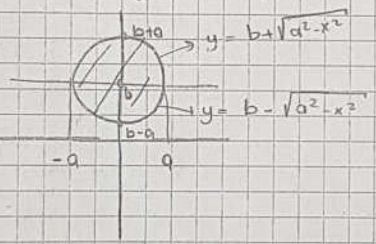
$$(x-0)^2+(y-b)^2=a^2$$

$$x^2+(y-b)^2=a^2$$

$$\sqrt{(y-b)^2}=\sqrt{a^2-x^2}$$

$$|y-b|=\sqrt{a^2-x^2}$$

$$y=b\mp\sqrt{a^2-x^2}$$



$$V=\pi\int_{-a}^{a}\left(b+\sqrt{a^2-x^2}\right)^2-\left(b-\sqrt{a^2-x^2}\right)^2dx$$

$$=\pi\int_{-a}^{a}\cancel{b^2}+2b\sqrt{a^2-x^2}+\cancel{a^2-x^2}-\left(\cancel{b^2}-2b\sqrt{a^2-x^2}+\cancel{a^2-x^2}\right)dx$$

$$V=\pi.\int_{-a}^{a}\left(4b\sqrt{a^2-x^2}\right)dx=4b\pi\int_{-a}^{a}\sqrt{a^2-x^2}\,dx.$$

$x=a.\sin t$
$dx=a.\cos t\,dt$

$$=4b\pi\int_{\alpha}^{\beta}\sqrt{a^2-a^2\sin^2t}\;a\cos t\,dt=4ba^2\pi\int_{\alpha}^{\beta}\cos^2t\,dt$$

$$V = 4a^2 b\pi \int_{\alpha}^{\beta} \cos^2 t \, dt \rightarrow 4a^2 b\pi \int_{\alpha}^{\beta} \left(\frac{1+\cos 2t}{2}\right) dt$$

$$= 4a^2 b\pi \left(\frac{t}{2} + \frac{\sin 2t}{4}\right) \Bigg|_{\alpha = -\pi/2}^{\beta = \pi/2}$$

$x = a \cdot \sin t$
$-a = a \cdot \sin t$
$\sin t = -1$
$t = -\frac{\pi}{2}$

$\frac{3\pi}{4}$ değil

Bu taraftan bakacağız.

$x = a \cdot \sin t$
$a = a \cdot \sin t$
$\sin t = 1$
$t = \pi/2$.

$$4a^2 b\pi \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right)$$

$$= 4a^2 b\pi \left(\frac{\pi}{2}\right) = 2a^2 b\pi^2 \text{ br}^3$$

Genelleştirilmiş (Has olmayan) İntegraller:

Belirli (reaman) integrali tanımlanırken hatırlanacağı gibi $[a,b]$ f fonksiyonu sınırlıdır (süreklidir) başka bir deyimle f fonksiyonunun bir aralıktaki integralinden bahsedebilmek için $[a,b]$ fonk. sınırlı ve dolayısıyla sürekli olması gerekir. Fakat pekçok fonk. $(-\infty, a]$, $[a, +\infty)$, $(-\infty, +\infty)$'da tanımlı olabilir. Aynı zamanda bazı fonksiyonlar düşey asimptotlara sahip old. sınırsızdır. Dolayısıyla reeman int. gibi hesaplama yapılamaz.

Örneğin,

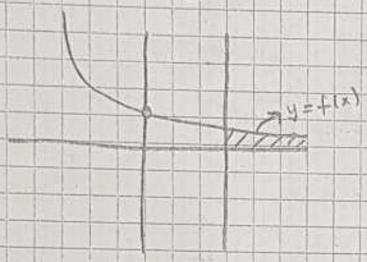
$y = e^{-x}$ eğrisi $x=1$ doğrusu ve x eks. arasında kalan böl. alanını hesaplamak için,

$$y = e^{-x} = \frac{1}{e^{x}}$$

$x \to +\infty \quad , \quad y \to 0 \quad$ (x ekseni)

$x \to -\infty \quad , \quad y \to +\infty$

$x \to 0 \quad , \quad y \to 1$



$$A = \int_{x=1}^{x=+\infty} f(x)\,dx$$

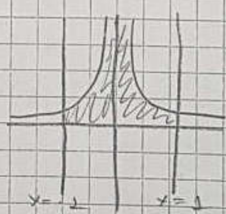
Örneğin,

$y = \frac{1}{x^2}$ eğrisi ile $x=1$ doğruları arasında kalan bölgenin alanını bulunuz.

$x=0$ düşey asimptot

$x = \mp\infty \quad y = 0$

$$\left.\begin{array}{l} x \to 0^+ \\ x \to 0^- \end{array}\right\} \; y = +\infty$$



$f(x) = \frac{1}{x^2}$ eğr. düşey asimptota sahiptir. Sınırsızdır.

$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

$x=0 \in [-1, 1]$ old. $f(x) = \frac{1}{x^2}$ olduğunda $[-1,1]$'de süreksizdir.

$\int_{-1}^{1} \frac{1}{x^2}$ belirli int. değil, genelleştirilmiş int.'dir

* Genelleştirilmiş integraller aralığın sınırsız, fonksiyonun sınırsız, hem aralığın sınırsız hem fonksiyonun sınırsız olmasıyla 3 türde incelenir.

Genelleştirilmiş İnt.

1. tip:

$f(x)$, $(-\infty, a]$, $[a, +\infty)$ ya da $(-\infty, +\infty)$ 'da sürekli,

$\int_{-\infty}^{1} \frac{dx}{x-2}$ $x=2 \notin (-\infty, 1]$, $f(x) \in (-\infty, 1]$ 'de sürekli.

$\int_{3}^{+\infty} \frac{dx}{(x+1)^4}$ $x=-2 \notin [3, +\infty)$, $f(x)$ $[3, +\infty)$ 'da sürekli

$\int_{-\infty}^{+\infty} \frac{dx}{x^2+1} > 0$ $f(x) = \frac{1}{1+x^2} \in (-\infty, +\infty)$ 'da sürekli.

2. tip:

$\int_{a}^{b} f(x)\,dx$

$f(x)$, $[a, b]$ 'de süreksiz

$\int_{0}^{3} \frac{dx}{x^4}$ $x=0 \in [0, 3]$ olduğundan süreksizlik var

$f(x)$, $[0, 3]$ 'de süreksiz

3. tip :

$$\int_{-\infty}^{+3} \frac{dx}{x-1} \qquad x=1 \in (-\infty, 3]$$

süreksiz

Tanım: a, bir reel sayı f, $\forall t \geq a$ $[a,t]$ integrallenebilir olsun.

$\lim\limits_{t \to +\infty} \int_a^t f(x)\,dx$ ifadesine f'in $[a,+\infty)$'da 1. tip genelleştirilmiş integrali denir ve $\int_a^{+\infty} f(x)\,dx = \lim\limits_{t \to +\infty} \int_a^t f(x)\,dx$'dir.

Benzer şekilde f fonksiyonu $(-\infty, b]$ ve $(-\infty, +\infty)$'da $f(x)$'in sürekli olması durumunda 1. tip genelleştirilmiş int. sahiptir.

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{+\infty} f(x)\,dx$$

$$\lim_{t_1 \to -\infty} \int_{t_1}^{c} f(x)\,dx + \lim_{t_2 \to +\infty} \int_{c}^{t_2} f(x)\,dx$$

e/ $\int_{1}^{+\infty} e^{-x}\,dx = ?$

$f(x) = e^{-x}$ üstel fonk her yerde sürekli

$\lim_{t \to +\infty} \boxed{\int_{1}^{t} e^{-x}\,dx}$

$= \lim_{t \to +\infty} \left(-e^{-x} \Big|_{x=1}^{x=t} \right)$

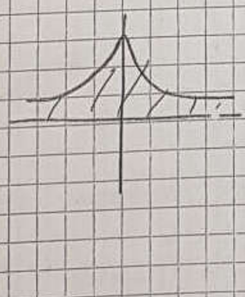
$= \lim_{t \to +\infty} \left(-e^{-t} - e^{-1} \right) = \lim_{t \to +\infty} \left(\frac{-1}{e^{t}} + \frac{1}{e} \right)$

$= \frac{1}{e}$

e/ $y = \frac{1}{1+x^2}$ eğrisiyle x eks. arasında kalan bölgenin alanı?

$x=0$ için $y=1$ $(0,1)$

$x \to \mp\infty$, $y \to 0$



$A = \int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx = \lim_{t \to +\infty} \int_{-t}^{+t} \frac{dx}{1+x^2} = \lim_{t \to +\infty} \arctan x \Big|_{x=-t}^{x=t}$

$\lim_{t \to +\infty} \left(\arctan t - \arctan(-t) \right)$

$= \arctan(+\infty) - \arctan(-\infty)$

$= \pi/2 - (-\pi/2) = \pi$ ✓

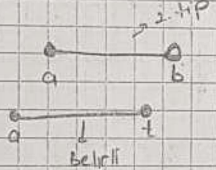
Tanım:

f fonksiyonu [a,b) aralığın herbir kapalı alt aralığı üzerinde integrallenebilir.

$\lim_{t \to b^-} f(x) = \mp \infty$ olsun. Bu takdirde

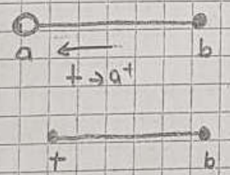
$\int_a^b f(x)\,dx$ (2. tip) genelleştirilmiş integral denir.

$$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$$ olarak hesaplanır.



[a, b)

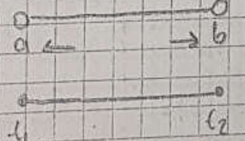
f fonksiyonu (a,b] 'nin herbir kapalı aralığında int. olsun.



$$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$$

f fonk. (a,b) 'nin herbir kapalı alt aralığında int. olsun.

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$$= \lim_{t_1 \to a^+} \int_{t_1}^{c} f(x)\,dx + \lim_{t_2 \to b^-} \int_{c}^{t_2} f(x)\,dx$$

(11) a/ $\displaystyle\int_0^1 \frac{dx}{\sqrt[4]{1-x}} = ?$

$1 - x = 0$

$x = 1$ singüler nokta

$x = 1 \in [0,1]$ 'de $f(x)$, $[0,1)$ 'de sürekli

(1. tip)

$0 \to 1$

$0 \quad t$

$$\lim_{t \to 1^-} - \left(\frac{(1-x)^{3/4}}{3/4} \right) \Bigg|_{x=0}^{x=t}$$

$$= - \lim_{t \to 1^-} \left(\underbrace{\frac{(1-t)^{3/4}}{3/4}}_{0} - \underbrace{\frac{(1-0)^{3/4}}{3/4}}_{4/3} \right) = 4/3.$$

11) Ör/

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2-4x+5}$$

$x^2-4x+5 = (x-2)^2+1 > 0$

$b^2-4ac = 16-4\cdot1\cdot5 < 0$

real kök yok.

paydayı sıfır yapan değer yok.

$$\int_{-\infty}^{+\infty} \frac{dx}{(x-2)^2+1} \overset{1.\ tip}{=} \lim_{t\to+\infty} \int_{-t}^{+t} \frac{dx}{(x-2)^2+1}$$

$$\lim_{t\to+\infty} \arctan(x-2)\Big|_{x=-t}^{x=t} = \lim_{t\to+\infty} \arctan(t-2) - \arctan(-t-2)$$

$$= \arctan(+\infty) - \arctan(-\infty)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi \checkmark$$

$(-\infty, +\infty)$ 'da sürekli

12) Ör/

$$\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = ?$$

$\sqrt{1-x^2} = 0$

$x=1$ singüler nokta

$x=1 \in [0,1]$ 'da.

$f(x) = \dfrac{\arcsin x}{\sqrt{1-x^2}}$, $[0,1]$ 'de süreksiz.

(2. tip)

$$\lim_{t\to1^-} \boxed{\int_0^t \frac{\arcsin x}{\sqrt{1-x^2}}\,dx} \to \text{örc}$$

$$\overset{u=\arcsin x,\ du=\frac{1}{\sqrt{1-x^2}}dx}{=} \lim_{t\to1^-} \int_\alpha^\beta u\,du = \frac{u^2}{2}\Big|_{u=\alpha}^{u=\beta}$$

$$\lim_{t\to1^-} \frac{(\arcsin x)^2}{2}\Big|_{x=0}^{x=t} \Rightarrow \lim_{t\to1^-} \frac{(\arcsin t)^2}{2} - \underbrace{\frac{(\arcsin 0)^2}{2}}_{0}$$

$$= \lim_{t\to1^-} \frac{(\arcsin 1)^2}{2} \to \left(\frac{\pi}{2}\right)^2 \cdot \frac{1}{2} = \frac{\pi^2}{8}$$

Ör/ $J = \int_{-1}^{1} \frac{dx}{1-x^2} = ?$

$1-x^2 = 0 \qquad 1 = x^2 \qquad x = \mp 1$ singüler nokta.

$x = \mp 1 \in [-1, +1]$ old. dan $f(x) = \frac{1}{1-x^2}$,

$[-1, 1]$ 'de süreksizdir 2. tip genelleştirilmiş int. ✓

$$\int_{-1}^{0} \frac{dx}{1-x^2} + \int_{0}^{1} \frac{dx}{1-x^2} = \underbrace{\lim_{t_1 \to -1^+} \int_{t_1}^{0} \frac{dx}{1-x^2}}_{I_1} + \underbrace{\lim_{t_2 \to 1^-} \int_{0}^{t_2} \frac{dx}{1-x^2}}_{I_2}$$

$$\int_{t}^{0} \frac{dx}{1-x^2} \longrightarrow \int_{t_1}^{0} \left(\frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x} \right) dx$$

$$\frac{1}{1-x^2} = \frac{A}{\underset{(1+x)}{1-x}} + \frac{B}{\underset{(1-x)}{1+x}}$$

$$= -\frac{1}{2} \ln|1-x| + \frac{1}{2} \ln|1+x| \Big|_{t_1}^{0}$$

$$1 = A(1+x) + B(1-x)$$

$$= \frac{1}{2} \ln\left|\frac{1+x}{1-x}\right| \Big|_{x=t_1}^{x=0}$$

$$1 = x\underbrace{(A-B)}_{0} + \underbrace{A+B}_{1}$$

$A = B = 1/2$

$$= \underbrace{\frac{1}{2} \ln 1}_{0} - \frac{1}{2} \ln\left|\frac{1+t_1}{1-t_1}\right|$$

$$\lim_{t \to -1^+} \int_{t}^{0} \frac{dx}{1-x^2} = \lim_{t_1 \to -1^+} \frac{-1}{2} \ln\left|\frac{1+t_1}{1-t_1}\right| = \frac{-1}{2} \ln\left|\frac{1-1}{1+1}\right|$$

$$= \frac{-1}{2} \ln|0|$$

$$= +\infty.$$

$\ln 1 = 0$
$\ln(+\infty) = +\infty$
$\ln(0) = -\infty$

$I = I_1 + I_2$.

I_1, $+\infty$ olduğundan

I int. $+\infty$ 'dir.

(ıraksak) int.

Uygulamalar:

1) $$\int_0^{\ln 5} \frac{\sqrt{e^x-1}}{1+3\cdot e^{-x}}\,dx = ?$$

$$e^x - 1 = u^2 \qquad e^x = 1+u^2$$

$$e^x\,dx = 2u\,du$$

$$dx = \frac{2u\,du}{1+u^2}$$

$$I = \int_\alpha^\beta \frac{u}{1+\frac{3}{1+u^2}} \cdot \frac{2u}{1+u^2}\,du = 2\int_\alpha^\beta \frac{u^2}{1+\frac{3}{1+u^2}} \cdot \frac{du}{1+u^2}$$

$$= 2\int_\alpha^\beta \frac{u^2}{1+u^2+3}\,du = 2\int_\alpha^\beta \frac{u^2+4-4}{u^2+4} \to 2\int_\alpha^\beta \frac{\cancel{u^2+4}}{\cancel{u^2+4}}\,du - 4\int_\alpha^\beta \frac{du}{u^2+4}$$

$$= 2\int_\alpha^\beta 1\,du - 4\int \frac{du}{u^2+4} \to 2u\Big|_\alpha^\beta - 8\arctan\left(\frac{u}{2}\right)\Big|_\alpha^\beta$$

$$= 2\sqrt{e^x-1}\Big|_{x=0}^{x=\ln 5} - 4\arctan\left(\frac{\sqrt{e^x-1}}{2}\right)\Big|_0^{\ln 5}$$

$$= 2(\sqrt{5-1}) - 4\cdot\arctan\left(\frac{\sqrt{5-1}}{2}\right)$$

$$= 2\cdot 2 - 4\arctan 1$$

$$= 4 - 4\cdot\frac{\pi}{4} = \underline{4-\pi}$$

2) $\displaystyle\int_0^4 \frac{\sqrt{x}+2}{x-4\sqrt{x}+5}\,dx = ?$

$x = u^2$
$dx = 2u\,du$

$$I = \int_\alpha^\beta \frac{u+2}{u^2-4u+5}\cdot 2u\,du \rightarrow 2\int_\alpha^\beta \frac{(u+2)u\,du}{u^2-4u+5} = 2\int_\alpha^\beta \frac{u^2+2u}{u^2-4u+5}\,du$$

$$\begin{array}{r|l} \cancel{u^2}+2u & u^2-4u+5 \\ -\cancel{u^2}+4u+5 & 1 \\ \hline 6u-5 & \end{array}$$

$$= 2\int_\alpha^\beta \left(1+\frac{6u-5}{u^2-4u+5}\right)du$$

$$= 2\int_\alpha^\beta 1\,du + 2\int_\alpha^\beta \frac{6u-5}{u^2-4u+5}\,du$$

$u^2-4u+5 = w$
$(2u-4)\,du = dw$

$$= 2\int_\alpha^\beta 1\,du + 2\int_\alpha^\beta \frac{3\left(2u-\frac{5}{3}\right)}{u^2-4u+5}\,du \rightarrow 2\int_\alpha^\beta du + 2\cdot 3\int_\alpha^\beta \frac{2u-4+4-\frac{5}{3}}{u^2-4u+5}\,du$$

$$= 2\int_\alpha^\beta du + 6\int_\alpha^\beta \frac{(2u-4)\,du}{u^2-4u+5} + 6\,\frac{7}{3}\int_\alpha^\beta \frac{du}{u^2-4u+5}$$

$$= 2u\Big|_\alpha^\beta + 6\int_{\alpha_1}^{\beta_1} \frac{dw}{w} + 14\int_\alpha^\beta \frac{du}{(u-2)^2+1}$$

$$= 2u\Big|_\alpha^\beta + 6\ln|w|\Big|_{\alpha_1}^{\beta_1} + 14\arctan(u-2)\Big|_\alpha^\beta$$

$u = \sqrt{x}$

$$= 2\sqrt{x}\Big|_{x=0}^{x=4} + 6\ln|x-4\sqrt{x}+5|\Big|_{x=0}^{x=4} + 14\arctan(x-2)\Big|_{x=0}^{x=4}$$

$$2\left(\sqrt{4}-0\right)+6\left(\ln\left|4-4\sqrt{4}+5\right|-\ln\left|0-4\cdot 0+5\right|\right)+14\left(\arctan\sqrt{4}\cdot 2\right)$$

3) $$\int_0^{1/2}\sqrt{1-x^2}\,dx=?$$

$$\sqrt{a^2-x^2}\Rightarrow x=a\sin t$$

$x=\sin t$
$dx=\cos t\,dt$ → trigonometrik ifade böyle yap

$x=0$ için $0=\sin t$, $t=0$ $1/2=\sin t$, $t=\pi/6$

$$I=\int_0^{\pi/6}\underbrace{\sqrt{1-\sin^2 t}}_{\cos t}\cdot\cos t\,dt \qquad I=\int_0^{\pi/6}\cos^2 t\,dt \rightarrow \int_0^{\pi/6}\frac{1+\cos 2t}{2}$$

$$\left(\frac{t}{2}+\frac{\sin 2t}{4}\right)\Bigg|_{t=0}^{t=\pi/6} \rightarrow \left(\frac{\pi/6}{2}+\frac{\sin \pi/3}{4}\right)-0$$

$$\frac{\pi}{12}+\frac{\sqrt{3}}{8}$$ ✓

4) $$\int_0^1 \arcsin\left(\frac{x}{2}\right)dx=?$$

$\frac{x}{2}=w$ $x=2w$, $dx=2\,dw$

$$2\int_\alpha^\beta \underbrace{\arcsin w}_{u}\,\underbrace{dw}_{dv}$$

$u=\arcsin w$ $dv=dw$
$du=\frac{1}{\sqrt{1-w^2}}$ $v=w$

$$2\left(u.v\Big|_\alpha^\beta-\int_\alpha^\beta v\,du\right)$$

$$=2\left(w\arcsin w\Big|_\alpha^\beta-\int_\alpha^\beta\frac{w}{\sqrt{1-w^2}}dw\right)$$

$1-w^2=t^2$
$-2w\,dw=2t\,dt$

$$=2\left(w.\arcsin w\Big|_\alpha^\beta-\int_{\alpha_1}^{\beta_1}\frac{-t\,dt}{\sqrt{t^2}}\right)$$

$$=2\left(\frac{x}{2}\cdot\arcsin\left(\frac{x}{2}\right)\Big|_{x=0}^{x=1}+\sqrt{1-\frac{x^2}{4}}\Big|_{x=0}^{x=1}\right)$$

$$=2\cdot\left[\frac{1}{2}\arcsin\frac{1}{2}-0+\sqrt{1-\frac{1}{4}}-1\right]$$
$$-\left[2\cdot\frac{1}{2}\cdot\frac{\pi}{6}+\frac{\sqrt{3}}{2}-1\right]$$

5) $$\int_{-1}^{+1} \frac{2x^3-7x+4}{\sqrt{x^2+3}}\, dx = ?$$

$$\int \frac{2x^3-7x+4}{\sqrt{x^2+3}}\, dx = (dx^2+ex+f) \cdot \sqrt{x^2+3} + \lambda \int \frac{dx}{\sqrt{x^2+3}}$$

$$\frac{2x^3-7x+4}{\sqrt{x^2+3}} = (2dx+e) \cdot \sqrt{x^2+3} + \frac{x}{\sqrt{x^2+3}} \cdot (dx^2+ex+f) + \frac{\lambda}{\sqrt{x^2+3}}$$

(1) $\qquad (\sqrt{x^2+3})$

$$2x^3-7x+4 = (2dx+e)(x^2+3) + dx^3 + ex^2 + fx + \lambda$$

$$2x^3-7x+4 = 2dx + 6dx + ex^2 + 3e + dx^3 + ex^2 + f(x) + \lambda$$

$$2x^3-7x+4 = \underbrace{3d}x^3 + 2ex^2 + x(6d+f) + 3e + \lambda$$

$3d = 2$ $\qquad 2e = 0$ $\qquad 6d+f = -7$ $\qquad 3e+\lambda = 4$

$d = 2/3$ $\qquad e = 0$ $\qquad 4+f = -7$ $\qquad \lambda = 4$

$f = -11$

$$\int_{-1}^{1} \frac{2x^3-7x+4}{\sqrt{x^2+1}}\, dx = \left(\frac{2}{3}x^2 - 11\right) \cdot x^2+3 \Bigg|_{x=-1}^{x=1} + 4 \int_{-1}^{1} \frac{dx}{\sqrt{x^2+3}}$$

$$\left(\frac{2}{3} - 11\right) \cdot 1+3 - \left(\frac{2}{3} - 11\right) \cdot (1+3) + 4 \cdot \int_{-1}^{1} \frac{dx}{\sqrt{x^2+3}}$$

$$= 4 \ln \left|x + \sqrt{x^2+3}\right| \Bigg|_{y=-1}^{x=1} = 4 \ln |1+2| - \underbrace{\ln |-1+2|}_{0}$$

$$= 4 \ln 3$$

(6) $y = x^2 + x - 6$ paraboli ile bu parabolün x eksenlerini kestiği noktalardan teğet geçen doğrular arasında kalan bölgenin alanı = ?

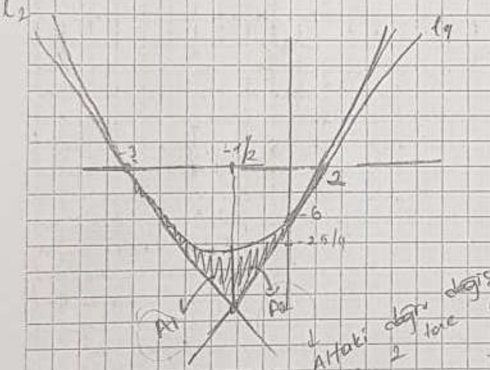
$x = 0 \quad y = -6 \quad A(0, -6)$

$y = 0 \quad (x+3)(x-2) \quad x_1 = -3 \quad (-3, 0)\ (2, 0)$

$x_2 = 2$

$T(x_0, y_0) = ?$

$x_0 = \frac{-1}{2} \quad y_0 = \frac{1}{4} - \frac{1}{2} - 6 = \frac{1}{4} - \frac{2-24}{4} \quad T\left(\frac{-1}{2}, \frac{-25}{4}\right)$



Alttaki doğru değiştiği için 2 tane için m

Teğet doğrusu denklemi !

$y - y_0 = m(x - x_0)$

l_2 için

$2x+1 \Big|_{x=-3} \rightarrow -5$

$y - 0 = -5(x+3)$

$y = -5x - 15 \rightarrow l_2$

l_1 için

$2x+1 \Big|_{x=2} = \boxed{5}$

$y - 0 = 5(x-2)$

$y = 5x - 10$

$A = A_1 + A_2$

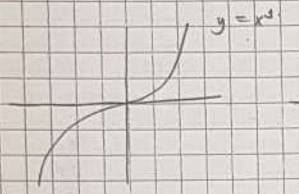
$$\int_{-3}^{-1/2} x^2 + x - 6 - (-5x - 15) + \int_{-1/2}^{2} (x^2 + x - 6) - (5x - 10)\,dx$$

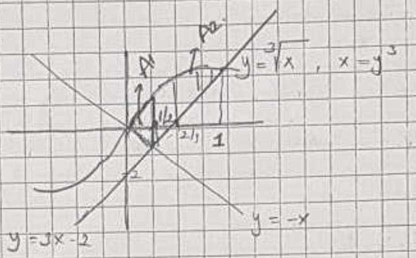
$-5x - 15 = 5x - 10$

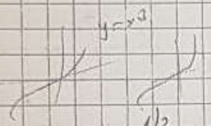
$10x = -5$

$x = -1/2$

7) $y = \sqrt[3]{x}$ eğrisi, $y = -x$ ve $y = 3x - 2$ doğruları arasında kalan bölgenin alanını bulunuz.







$-x = 3x - 2$
$4x = 2$
$x_1 = 1/2$

$\sqrt[3]{x} = 3x - 2$
$y = 3x - 2$
$3y^3 - 2$
$y = 1$
$1 = 3 \cdot 1^3 - 2 \rightarrow x_2 = 1$

$$\int_0^{1/2} \left(\sqrt{x^3} - (-x)\right) dx + \int_{1/2}^{1} \left(\sqrt{x^3} - (3x - 2)\right) dx$$

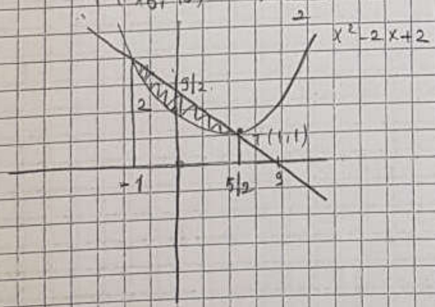
$$\left(\frac{x^{4/3}}{4/3} + \frac{x^2}{2}\right)\Bigg|_0^{1/2} + \left(\frac{x^{4/3}}{4/3} - \frac{3x^2}{2} + 2x\right)\Bigg|_{1/2}^{1}$$

8) $y = x^2 - 2x + 2$ eğrisi ile $x + 2y - 9 = 0$ doğrusu arasında kalan bölgenin alanını bulunuz.

$x = 0 \quad y = \frac{9}{2}$

$x = 0 \quad y = 2 \quad (0, 2)$
$y = 0 \quad (x-1)^2 + 1 \neq 0 \quad$ x'i kesmez.

$T(x_0, y_0) \rightarrow \frac{2}{2} = 1 \qquad 1 - 2 + 2 = 1 \qquad (1, 1)$



$x^2 - 2x + 2 = \frac{9 - x}{2}$

$2x^2 - 4x + 4 = 9 - x$
$2x^2 - 3x - 5 = 0$

$2x$	-5
x	$+1$

$x_1 = 5/2$
$x_2 = -1$

$$\int_{-1}^{5/2} \frac{9-x}{2} - (x^2-2x+2)\,dx \rightarrow \left.\frac{9}{2}x - \frac{x^2}{2} - \frac{x^3}{3} + x^2 - 2x \right|_{x=-1}^{x=5/2}$$

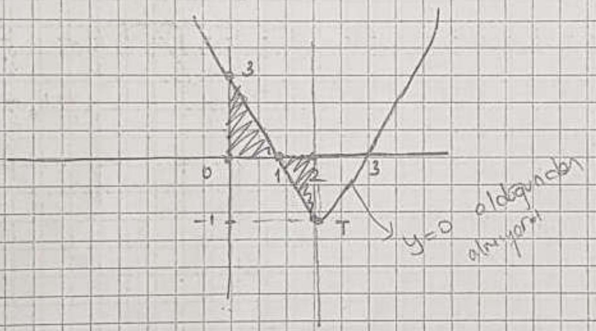
9) $y = x^2-4x+3$ eğrisi ile $y=0$, $x=0$ $x=2$ doğruları arasında kalan bölgenin alanını bulunuz

$y = x^2-4x+3$

$x=0$ $y=3$ $(0,3)$

$y=0$ $(x-3)(x-1)$ $(3,0)$ $(1,0)$

$T(x_0, y_0) \rightarrow \frac{4}{2} = 2$ $4-8+3 = -1$ $(2,-1)$



$$\int_0^1 (x^2-4x+3)-0\,dx + \int_1^2 0-(x^2-4x+3)\,dx$$

$$\left.\frac{x^3}{3} - \frac{4x^2}{2} - 3x\right|_0^1 - \left.\left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right)\right|_1^2$$

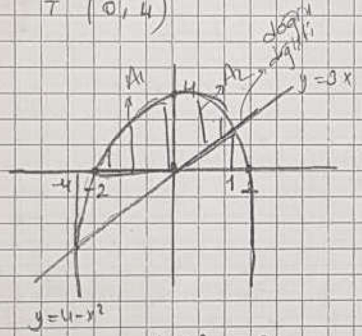
10) a) $y = 4 - x^2$ parabolü ile $y \geq 0$, $y \geq 3x$ eşitsizlikleri arasında kalan bölgenin alanını bulunuz.

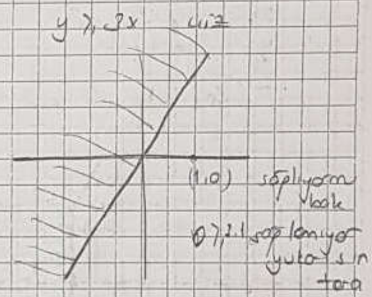
b) x eksi etrafında döndürülmesiyle meydana gelen dönel cismin hacmi?

a) $x = 0 \quad y = 4 \quad (0,4)$

$y = 0 \quad x_{12} = \mp 2 \quad (2,0) \quad (-2,0)$

$T \ (0,4)$





$4 - x^2 = 3x$

$x^2 + 3x - 4 = 0$

$x_1 = -4 \ , \ x_2 = 1$

$$\int_{-2}^{0} (4 - x^2 - 0)\,dx \quad + \quad \int_{0}^{1} (4 - x^2) - (3x)\,dx$$

$$b)\ V = \pi \int_{-2}^{0} (4 - x^2)^2\,dx \ + \ \pi \int_{0}^{1} (4 - x^2)^2 - (3x)^2\,dx$$

11) $\int_0^{+\infty} x \cdot e^{-x^2}\, dx = ?$

$x \cdot e^{-x^2} \in C\ [0, +\infty)$ sürekli

$\underset{P}{\downarrow}\ ,\ \underset{''}{\downarrow}$

$I = \lim_{t \to +\infty} \int_0^t x \cdot e^{-x^2}\, dx \rightarrow$ Belirli int. oldu

$x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$\frac{1}{2} \lim_{t \to +\infty} \int_\alpha^\beta du \cdot e^{-u} \rightarrow \frac{1}{2} \lim_{t \to +\infty} -e^{-u} \Big|_\alpha^\beta \rightarrow \frac{-1}{2} \lim_{t \to +\infty} \left(e^{-x^2}\right) \Big|_{x=0}^{x=t}$

$= \frac{-1}{2} \lim_{t \to +\infty} \left(e^{-t^2} - e^0\right)$

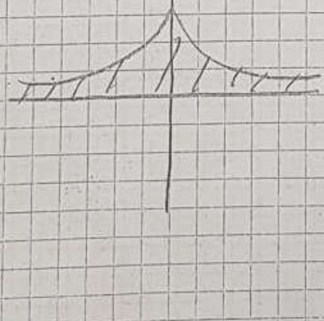
$= \frac{-1}{2} \lim_{t \to +\infty} \left(\cancel{\frac{1}{e^{t^2}}} - 1\right) = \frac{1}{2}$

12) $y = \frac{1}{1+x^2}$ eğrisi ile x ekseni arasında kalan bölgenin alanını hesaplayınız

$x = 0 \quad y = 1$

$x \to \mp\infty \quad y = 0 \rightarrow x$ eksenine yaklaştı demek

$y \neq 0 \quad x$ eksenini kesmez



$\int_{-\infty}^{+\infty} \left(\frac{1}{1+x^2} - 0\right) dx$

$(-\infty, +\infty)$'da sürekli

$\lim_{t \to +\infty} \int_{-t}^{+t} \frac{1}{1+x^2}\, dx \rightarrow \lim_{t \to +\infty} \arctan x \Big|_{-t}^{+t}$

$\lim_{t \to +\infty} \arctan t - \arctan(-t)$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$

$$\int_2^{+\infty} \frac{\ln x}{x}\,dx = ? \qquad x=0 \notin [2,+\infty)$$

$$\lim_{t\to+\infty} \int_0^t \frac{\ln x}{x}\,dx \;\to\; \underset{\substack{\ln x = u \\ \frac{1}{x}dx = du}}{=} \lim_{t\to+\infty} \int_\alpha^\beta u\,du$$

$$\lim_{t\to+\infty} \frac{u^2}{2}\Big|_\alpha^\beta \;\to\; \lim_{t\to+\infty} \frac{\ln^2 x}{2}\Big|_2^t$$

$$\lim_{t\to+\infty} \underbrace{\frac{\ln^2 t}{2}}_{+\infty} - \frac{\ln^2 2}{2} = +\infty$$

int. yakınsak değildir.
ve değeri yoktur.

13) $$\int_0^{\pi/4} \sin^5 x \cdot \cos^7 x\,dx \;? \;\to\; \int_0^{\pi/4} \sin^5 x \cdot \cos^6 x \cdot \cos x\,dx$$

tek

$$\int_0^{\pi/4} \sin^5 x \cdot (\cos^2 x)^3 \cos x\,dx \;\underset{\substack{\sin x = u \\ \cos x\,dx = du}}{=} \int_\alpha^\beta u^5 (1-u^2)^3\,du$$

$$= \int_\alpha^\beta u^5 (1 - 3u^2 + 3u^4 - u^6)\,du \;\to\; = \int_\alpha^\beta (u^5 - 3u^7 + 3u^9 - u^{11})\,du$$

$$= \frac{u^6}{6} - \frac{3u^8}{8} + \frac{3u^{10}}{10} - \frac{u^{12}}{12}\Big|_\alpha^\beta =$$

$$\frac{\sin^6 x}{6} - \frac{3\sin^8 x}{8} + \frac{3\sin^{10} x}{10} - \frac{\sin^{12} x}{12}\Big|_0^{\pi/4}$$

14) a) $\int_3^4 \frac{6x+8}{x^2+x-6}\,dx$ $(x^2+x-6) \to (x+3)(x-2)$ $x_1=-3$ $x_2=2$ $[-3,2] \notin [3,4]$

Belirli integral

b) $\int_2^4 \frac{6x+8}{x^2+x-6}\,dx$ $x_2 = 2 \in \underbrace{[2,4]}_{\text{sonlu bir aralık}}$ $f(x) \notin [2,4]$ 2. tip gen. int.

a) $\frac{6x+8}{(x+3)(x-2)} = \frac{A}{\underset{(x-2)}{x+3}} + \frac{B}{\underset{(x+3)}{x-2}}$

$6x+8 = Ax-2A+Bx+3B$

$$\begin{array}{l} 2/\ A+B=6 \\ -2A+3B=8 \\ \hline 2A+2B=12 \\ -2A+3B=8 \\ \hline 5B=20 \end{array}$$

$B=4$
$A=2$

$\int_3^4 \left(\frac{2}{x+3} + \frac{4}{x-2}\right) dx \Rightarrow 2\ln|x+3| + 4\ln|x-2| \Big|_3^4$

$\ln(x+3)^2 \cdot \ln(x-2)^4 \Big|_3^4$

$= \ln|7^2 \cdot 2^4| - \ln|6^2 \cdot 1|$

$= \ln\left|\frac{7^2 \cdot 16}{36}\right| = \ln\left|\frac{49 \cdot 4}{9}\right|$

b) 2 ——— 4, $\underline{2+a}$ ——— 4

$\int_2^4 \frac{6x+8}{x^2+x-6}\,dx = \lim_{a \to 0^+} \underbrace{\int_{2+a}^4 \frac{6x+8}{x^2+x-6}\,dx}_{\text{Belirli integral}}$

$$\lim_{a\to 0^+} \ln |x+3|^2 \cdot |x-2|^4 \Big|_{2+a}^{4}$$

$$= \lim_{a\to 0^+} \ln|49 \cdot 16| - \ln|\underbrace{5+a|^2 \cdot a^4}_{0}$$ $\ln 0 = -\infty$

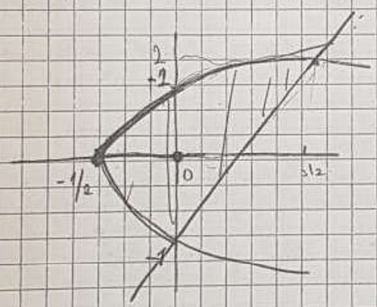
$= +\infty$

(5) $y^2 = 2x+1$, $y = 2x-1$ } Alanını bul.

$y^2 = 2x+1$

$x=0 \quad y=\mp 1 \quad (0,1) \quad (0,-1)$

$y=0 \quad x=-1/2 \quad (-1/2, 0)$



$$\int_{-1/2}^{0} \left(\sqrt{2x+1} - \sqrt{2x+1}\right) dx + \int_{0}^{3/2} \sqrt{2x+1} - (2x-1)\, dx$$

x'e göre 2 türlü

$y+1 = y^2-1$

$y^2 - y - 2 = 0$

$(y-2)(y+1)$

$y_1 = -1 \quad y_2 = 2$

$$A = \int_{-1}^{2} \left[\frac{y+1}{2} - \frac{y^2-1}{2}\right] dy$$

(11) $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}$ eğrisinin $1 \leq x \leq 3$ noktaları arasında kalan parçasının uzunluğu.

$$\int_1^3 \sqrt{1+(y')^2}\, dx \qquad y' = x - \frac{1}{4x}$$

$$\int_1^3 \sqrt{1+\left(x-\frac{1}{4x}\right)^2}$$

$$1 + x^2 - 2 \cdot x \cdot \frac{1}{4x} + \frac{1}{16x^2}$$

$$\underbrace{x^2}_{(x)^2} + \frac{1}{2} + \underbrace{\frac{1}{16x^2}}_{\left(\frac{1}{4x}\right)^2}$$

$$\int_1^3 \sqrt{\left(x+\frac{1}{4x}\right)^2}$$

$$\frac{x^2}{2} + \frac{1}{4}\ln x \Big|_1^3$$

$$\frac{x^2}{2}+\frac{\ln x}{4}\Big|_1^3 \rightarrow \left(\frac{9}{2}+\frac{\ln 3}{4}\right)-\left(\frac{1}{2}+\frac{\ln 1}{4}\right)$$

$$\Rightarrow \frac{\ln 3+16}{4}$$

Kutupsal Kordinatlarda Verilen Eğrilerin Grafiklerinin Çizimi,

$r=f(\theta)$ eşitliği ile verilen bir eğriyi çizmek için aşağıdaki yollar izlenir.

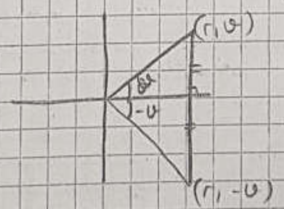
1) Tanım kümesi bulunur.

2) f Periyodik bir fonksiyon ise, periyodu bulunur. Periyot T ise T uzunluğundaki bir aralık üzerinde inceleme yapmak yeterlidir. Çünkü tüm aralıklarda fonksiyonun aldığı değ fonksiyonun değerleriyle aynıdır.

3) Simetri araştırılır bu simetrinin birkaç türesi şu şekildedir a) (r,θ) şek verilen denklem için $(r,-\theta)$ veya $(-r,\pi-\theta)$ denklemi sağlıyor ise eğri x eks. göre simetriktir.

Örneğin, $r=1+\cos\theta$

$r=1+\cos(-\theta)=1+\cos\theta$



b) (r, θ) denklemi sağlandığında, $(-r, -\theta)$ veya $(r, \pi - \theta)$ denk. sağlıyorsa eğri y eksenine göre simetridir.

Örneğin, $\sin(\theta)$ fonksiyonunu içeren eğriler, y eksenine göre simetriktir.

Eğrinin simetri özelliğini sağlaması eğrinin incelendiği T periyot aralığını daraltır. Örneğin $\cos\theta$ fonk. içeren kutupsal bir eğrinin periyodu $T = \frac{2\pi}{1} \Rightarrow 2\pi$'dir. Normalde incelenmesi gereken aralık $[-\pi, \pi]$ olması gerekirken eğri x eksenine göre simetri olduğundan incelenecek aralık $(0, \pi)$ olarak alınır. Benzer şekilde $\sin\theta$ fonk. içeren eğri için periyot $\rightarrow T = 2\pi$'dir fakat eğri y eksine göre simetri old. normalde incelenmesi gereken $[-\pi, \pi]$ iken $\left[\frac{-\pi}{2}, \frac{+\pi}{2}\right]$ olmuştur.

4) $r' = f'(\theta)$ türevin işareti incelenerek eğrinin kutba nerede yaklaştığını veya uzaklaştığını saptarız.

5) Fonksiyonun inceleme aralığında özel noktalarda aldığı değerler hesaplanır.

6) Değişim tablosu yapılır. Bulunan değerler bu tabloda belirtilir.

7) Değişim tablosuna göre çizim yapılır, simetri söz konusu ise simetriler alınarak çizim tamamlanır.

Her yerde sürekliler $\begin{cases} \text{polinom} \\ \text{üstel} \\ \text{Trigonometrik} \end{cases}$

ÖR/ $r = 2(1+\sin\theta)$ eğrisini çiziniz.

1) $T.K = I.R$

2) $T = 2\pi$ $[-\pi, \pi]$ incelenmesi gereken aralık. Ama eğri Oy eksi göre paralel olduğundan $\left[\frac{-\pi}{2}, \frac{+\pi}{2}\right]$'de inceleme yapılır.

3) $\theta = \frac{-\pi}{2}$ için $r = 2\left(1+\sin\left(\frac{-\pi}{2}\right)\right) = 2\left(1-\overset{1}{\overline{\sin\frac{\pi}{2}}}\right) = 0$

$\theta = \frac{-\pi}{6}$ için $r = 2\left(1+\sin\left(\frac{-\pi}{6}\right)\right) = 2\left(1-\sin\frac{\pi}{6}\right) = 2 \cdot \frac{1}{2} = 1$

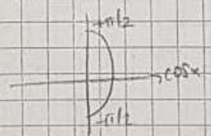
$\theta = 0$ $r = 2(1+\sin 0) = 2$

$\theta = \frac{\pi}{6}$ $r = 2\left(1+\sin\frac{\pi}{6}\right) = 2\left(1+\frac{1}{2}\right) = 3$

$\theta = \frac{\pi}{2}$ $r = 2\left(1+\sin\frac{\pi}{2}\right) = 2(1+1) = 4$

4) $r' = 2\cos\theta$ $\theta \in \left[\frac{-\pi}{2}, \frac{+\pi}{2}\right]$

$\theta = \frac{-\pi}{2}, \frac{+\pi}{2}$ $\underline{\underline{r' = 0}}$

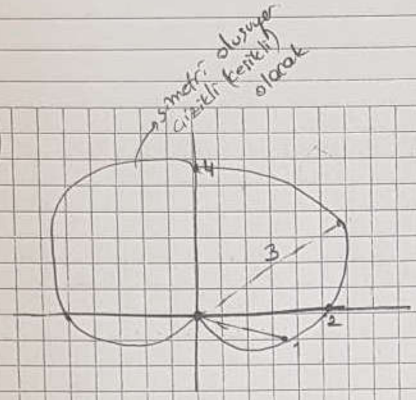


$\theta \neq \frac{-\pi}{2}, \frac{+\pi}{2}$ $\underline{r' > 0}$

5)

	$-\pi/2$		$-\pi/6$		0		$\pi/6$		$\pi/2$
r'	0	+		+		+		+	0
r	0	↗	1	↗	2	↗	3	↗	4

6)



4/2/ $r = 2 + 4\cos\theta$

1) $T.K = \mathbb{R}$

2) $T = 2\pi$. Normalde $[-\pi, +\pi]$ olması gerekirken Ox eksenine simetri olduğundan incelenecek aralık $[0, \pi]$'dir.

3) $\theta = 0$ için $r = 2 + 4\cos\theta = 6$

$\theta = \frac{\pi}{3}$ için $r = 2 + 4\cos\frac{\pi}{3} = 2 + 4\cdot\frac{1}{2} = 4$

$\theta = \frac{\pi}{2}$ için $r = 2 + 4\underbrace{\cos\frac{\pi}{2}}_{0} = 2$

$\theta = \frac{2\pi}{3}$ için $r = 2 + 4\cos\frac{2\pi}{3} = 2 + 4\cos 120$

$= 2 + 4(-\cos 60)$

$= 2 - 4\cdot\frac{1}{2} = 0$

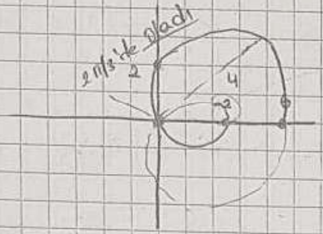
4) $r' = -4\sin\theta < 0$ $[0, \pi]$'de

5)

θ	0		$\pi/3$		$\pi/2$		$2\pi/3$		π
r'	0	-		-		-		-	0
r	6		4		2		0		-2

$-2 \to$ olduğundan tam terse gidiyor (eğri de)

6)



Ö/ $r = 2\cos\vartheta + 2\sin\vartheta$

(çember denklemi)

Kutupsal Kordinatlarda verilmiş olan bir eğrinin, alan hesabı,

f sürekli bir fonksiyon $r = f(\vartheta)$ formülüyla verilmiş olan bir eğrinin $\vartheta_1 \leq \vartheta \leq \vartheta_2$ açıları arasında kalan bölgenin alanı $A = \frac{1}{2}\int_{\vartheta_1}^{\vartheta_2} r^2(\vartheta)\, d\vartheta$ şeklinde hesaplanır.

Ö/ $r = a.(1+\cos\vartheta)$ kardiyoid eğrisinin alanını hesaplayınız.

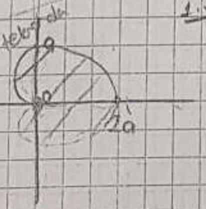
$[0,\pi]$ 'de inceleyelim

1. yol $\frac{A}{2} = \frac{1}{2}\int_0^{\pi} r^2(\vartheta)\, d\vartheta$

$A = \int_0^{\pi} a^2(1+2\cos\vartheta+\cos^2\vartheta)\, d\vartheta$

2. yol

4'ye kadar geçdi π'ye geldi tekrar da



$\frac{1}{2}\int_0^{2\pi} r^2(\vartheta)\, d\vartheta$

$$= \int_0^{\pi} a^2 \left(1 + 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= a^2 \left(\theta + 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_{\theta=0}^{\theta=\pi} = a^2 \left(\pi + 2\sin\pi + \frac{\pi}{2} + \frac{\sin 2\pi}{4} - 0\right)$$

$$= a^2 \frac{3\pi}{2} = \frac{3a^2\pi}{2} \text{ br}^2$$

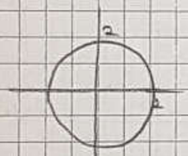
Ör/ $r=2$ çemberinin içinde $r = 2(1+\cos\theta)$ kardiyodinin dışında kalan bölgenin alanını bulunuz

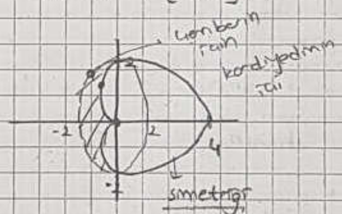
$r^2 = 4$ $\qquad\qquad$ $x^2+y^2=4$

$x^2+y^2=4$

$M(0,0)$ $r=2$ yarıçaplı çember

$T=2\pi \quad [0,\pi]$





$$A = \frac{1}{2}\int_{\pi/2}^{3\pi/2} 2^2 - \left(2(1+\cos\theta)\right)^2 d\theta = \frac{1}{2}\int_{\pi/2}^{3\pi/2} 4 - 4(1+2\cos\theta+\cos^2\theta)$$

$$= \frac{1}{2}\int_{\pi/2}^{3\pi/2} (-8\cos\theta - 4\cos^2\theta)\, d\theta \Rightarrow \frac{1}{2}\left(-8\sin\theta - \cancel{4}^{2}\left(\frac{1+\cos 2\theta}{\cancel{2}}\right)\right)\Bigg|_{\pi/2}^{3\pi/2}$$

$$\frac{1}{2}\left(-8\sin\theta - 2 - 2\cos 2\theta\right)\Bigg|_{\pi/2}^{3\pi/2}$$

Kutupsal Kordinatlarda Verilen Yay Uzunluğu

Kartezyen kordinatlarda, $y=f(x)$ veya $x=g(y)$ eğrilerinin uzunluk formülü $\ell=\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ veya $\ell=\int_{y_1}^{y_2}\sqrt{1+(g'(y))^2}\,dy$ şeklinde bulunur.

$$\left.\begin{aligned} x&=x(t)\\ y&=y(t)\end{aligned}\right\}\quad t_1\leq t\leq t_2$$

parametrik denklemiyle verilen eğrinin yay uzunluğu hesabı,

$$\ell=\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$$

şeklinde yazılır.

$r=f(\theta)$ kutupsal biçimde verilmiş eğrinin $\theta_1\leq\theta\leq\theta_2$ olmak üzere yay uzunluğu $\int_{\theta_1}^{\theta_2}\sqrt{r^2(\theta)+(r'(\theta))^2}\,d\theta$

Ör/ $r=a(1+\cos\theta)$ kardiyoid eğrisinin çevre uzunluğunu bulunuz.

$r'=-a\sin\theta$

$$r^2(\theta)+(r'(\theta))^2=a^2(1+2\cos\theta+\cos^2\theta)+a^2\sin^2\theta$$

$$a^2(1+2\cos\theta+\underbrace{\cos^2\theta+\sin^2\theta}_{1})$$

$$=2a^2(1+\cos\theta)$$

$$=2a^2\left(1+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)$$

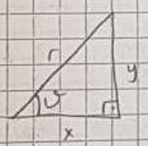
$$=2a^2\left(2\cos^2(\theta/2)\right)\rightarrow 4a^2\cos^2\frac{\theta}{2}$$

$$\ell = \int_0^{2\pi} \sqrt{r^2 + (r')^2}\, d\vartheta \Rightarrow 2 \int_0^{\pi} \sqrt{4a^2 \cos^2 \frac{\vartheta}{2}}\, d\vartheta$$

$$= 2 \cdot 2a \int_0^{\pi} \cos \frac{\vartheta}{2}\, d\vartheta = 4a \left. \frac{\sin \frac{\vartheta}{2}}{\frac{1}{2}} \right|_{\vartheta=0}^{\vartheta=\pi}$$

$$= 8a \Big(\underbrace{\sin \frac{\pi}{2}}_{1} - \sin 0 \Big) = 8a$$

11) $r = 1 + \cos\vartheta$ kardiyodinin $r = 3 \cdot \cos\vartheta$ çemberinin dışında kalan parçasının uzunluğu?



$\cos\vartheta = \frac{x}{r}$

$\sin\vartheta = \frac{y}{r}$

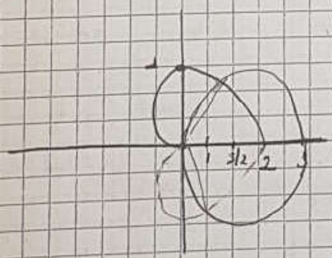
$r = 3 \cdot \frac{x}{r}$ $\quad r^2 = 3x$

$x^2 + y^2 = 3x \Rightarrow x^2 - 3x + y^2 = 0$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + y^2 = 0$

$\left(x - \frac{3}{2}\right)^2 + y^2 = \left(\frac{3}{2}\right)^2$

$M\left(\frac{3}{2}, 0\right) \quad r = \frac{3}{2}$



Kesişim

$1 + \cos\vartheta = 3\cos\vartheta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\vartheta_1 = \pi/3$

?

$\vartheta_2 = 2\pi - \pi/3 = 5\pi/3$

$\cos(5\pi/3)$

$\cos 300$

$\cos(360 - 60)$

$\cos 60 = 1/2$

$$r^2 + (r')^2 = (1 + \cos\vartheta)^2 + (-\sin\vartheta)^2$$

$$= 1 + 2\cos\vartheta + \underbrace{\cos^2\vartheta + \sin^2\vartheta}_{1}$$

$$= 2(1 + \cos\vartheta) = 2\left(1 + \cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2}\right)$$

$$= 4\cos^2\frac{\vartheta}{2}$$

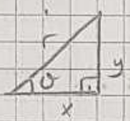
$$l = \int_{\pi/3}^{5\pi/3} \sqrt{r^2(\theta) + (r'(\theta))^2}\, d\theta = \int_{\pi/3}^{5\pi/3} \sqrt{4\cos^2\frac{\theta}{2}}\, d\theta = 2\int_{\pi/3}^{5\pi/3} \cos\left(\frac{\theta}{2}\right) d\theta$$

$$= 2\, \frac{\sin\frac{\theta}{2}}{\frac{1}{2}} \Bigg|_{\theta=\pi/3}^{\theta=5\pi/3} = 4$$

w2 / $r=1$ çemberinin dışında $r=2\sin\theta$ çemberinin içinde kalan bölgenin alanını hesaplayınız.

$r^2 = 1$

$x^2+y^2 = 1$



$\sin\theta = \frac{y}{r}$

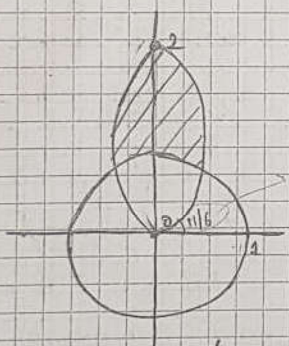
$r = 2\,\frac{y}{r}$ $\quad r^2 = 2y$

$x^2+y^2 = 2y$

$x^2 - 2y + y^2 = 0$

$x^2 + (y-1)^2 = 1$

$M(0,1)$ $r=1$ yarıçaplı çember



$$\frac{A}{2} = \frac{1}{2}\int_{\theta_1=\pi/6}^{\theta_2=\pi/2} r^2(\theta)\, d\theta = \int_{\pi/6}^{\pi/2} \left((2\sin\theta)^2 - 1^2\right) d\theta = \int_{\pi/6}^{\pi/2} (4\sin^2\theta - 1)\, d\theta$$

$$\int_{\pi/6}^{\pi/2} \cancel{4}^{2}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right) - 1\; d\theta = \int_{\pi/6}^{\pi/2} (2 - 2\cos 2\theta - 1)\, d\theta$$

$$= \int_{\pi/6}^{\pi/2} (1 - 2\cos 2\theta)\, d\theta$$

$$A = \theta - 2\,\frac{\sin 2\theta}{2} \Big|_{\pi/6}^{\pi/2}$$

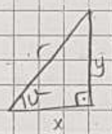
$x = r \cdot \cos\theta$
$y = r \cdot \sin\theta$

Ör/ Kutupsal kordinatlara geçerek $(x^2+y^2)^3 = 4x^2y^2$ eğrisi tarafından sınırlanan bölgenin alanını bulunuz.

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$x^2+y^2 = r^2$



$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

periyod 2π ol.
$\sin\{-\pi, +\pi\}$

periyod $[\pi]$ olduğundan
$\sin\left[\frac{-\pi}{2}, \frac{+\pi}{2}\right]$ olur.

$(r^2)^3 = 4r^2\cos^2\theta \cdot r^2\sin^2\theta$

$r^6 = 4r^4\cos^2\theta \cdot \sin^2\theta$

$r^2 = 4\cos^2\theta \cdot \sin^2\theta$

$r = 2\cos\theta\,\sin\theta$

$r = \sin 2\theta \qquad T = \frac{2\pi}{2} = \pi$

$\sin^2 2\theta$

$$\frac{1}{2}\int \sin^2 2\theta$$

$$\frac{1}{2}\int_{-\pi/4}^{\pi/4} \frac{1-\cos 4\theta}{2}$$

$$A = \frac{1}{2}\int r^2\theta\,(d\theta)$$

$\left[\frac{-\pi}{2}, \frac{+\pi}{2}\right]$, $\left[\frac{-\pi}{4}, \frac{+\pi}{4}\right]$ olacak

LAPLACE dönüşümleri ve uygulamaları:

Laplace dönüşümü, bir integral dönüşümü olup, fizik, mühendislik önemli yer alır. Laplace dön. esas amacı başlangıç koşulları ile verilen diferansiyel denklemin

$\boxed{L(3) = \frac{3}{s}} \rightarrow s>0$ kabul edilir ✓

$$L(f(t)) = \int_0^{+\infty} e^{-s.t}\, f(t)\, dt \neq \infty$$ olması lazım.

genel çözümünü bulurken keyfi sabitlerle uğraşmadan diferansiyel denklemin direk çözümüne ulaşılır.

Önerme α, β birer skaler $f(t)$ fonksiyonunun Laplace dönüşümü $(L(f(t)) = F(s))$ ve $g(t)$ dön. Laplace dön.

$g(t)$; $L(g(t)) = G(s)$ olsun.

$$L(\alpha . f(t) \mp \beta . g(t)) = \alpha . L(f(t)) \mp \beta . L(g(t))$$

Önerme $L(f(t)) = F(s) \iff f(t) = L^{-1}(F(s))$

Ör: $f(t) = c$ için $\underset{f(t)}{L(c)} = ?$

$$L(c) = \int_0^{+\infty} e^{-s.t}\, c\, dt = c \int_0^{+\infty} e^{-s.t}\, dt = c \lim_{u \to +\infty} \int_0^{u} e^{-s.t}\, dt$$

s>0 olmalı ki aşağıdaki değerde $u \to +\infty$ 0 olması lazım Laplace olmaz.

$$c \lim_{u \to +\infty} \left(\frac{e^{-s.t}}{-s} \right) \Bigg|_{t=0}^{t=u} = c \lim_{u \to +\infty} \left(\frac{e^{-s.u}}{-s} - \left(\frac{e^{-s.0}}{-s} \right) \right)$$

$$L(c) = \frac{c}{s}$$

$$\int_0^{\infty} e^{-s.t} f(t)\, dt$$

(1)

$$L\left(e^{\alpha \cdot t}\right)=\int_0^{+\infty} e^{-s \cdot t} \cdot e^{\alpha \cdot t}\, dt=\int_0^{+\infty} e^{t(\alpha-s)}\, dt$$

$$\lim_{u\to+\infty} \int_0^u e^{t(\alpha-s)}\, dt=\lim_{u\to+\infty}\left(\frac{e^{t(\alpha-s)}}{\alpha-s}\right)\Bigg|_{t=0}^{t=u}$$

$$=\lim_{u\to+\infty}\left(\frac{e^{u(\alpha-s)}}{\alpha-s}-\frac{e^{0\cdot(\alpha-s)}}{\alpha-s}\right)=\frac{1}{s-\alpha} \quad (s>\alpha)$$

$$L\left(3-e^{-4t}\right)=L(3)-L\left(e^{-4t}\right)=\frac{3}{s}-\frac{1}{s-(-4)}=\frac{3}{s}-\frac{1}{s+4}$$

(2)

$$L(\sin at)=\int_0^{+\infty} e^{-s \cdot t} \cdot \sin at\, dt=\lim_{w\to+\infty} \boxed{\int_0^{u} e^{-s \cdot t} \cdot \sin at\, dt}$$

Belirli kısmi uygulanırsa

$$\lim_{w\to+\infty}\left(u \cdot v\Big|_{t=0}^{t=w}-\int_{t=0}^{t=w} v\, du\right)$$

$\sin at=u$

$a\cos at\, dt=du$

$\int e^{-s \cdot t}\, dt=\int dv$

$\frac{e^{-s \cdot t}}{-s}=v$

$$\lim_{w\to+\infty} \frac{-1}{s}\sin at \cdot e^{-s \cdot t}\Big|_{t=0}^{t=w}+\frac{a}{s}\int_{t=0}^{t=w} e^{-s \cdot t} \cos at\, dt$$

$$\lim_{w\to+\infty}\left(\underbrace{\frac{-1}{s}\sin aw \cdot e^{-s \cdot w}}_{|\sin aw| \leq 1}+\underbrace{\frac{-1}{s}\sin(a \cdot 0) \cdot e^{-s \cdot 0}}_{0}+\frac{a}{s}\int_{t=0}^{t=w} e^{-s \cdot t} \cos at\, dt\right)$$

$e^{-s \cdot w}\to 0 \quad (w\to\infty,\ s>0)$

sınırlı . sıfıra giden fark $=0$

$$=\frac{a}{s}\lim_{w\to+\infty}\int_0^w \underbrace{e^{-s \cdot t} \cos at\, dt}$$

$\cos at=u$

$-\sin at \cdot a\, dt=du$

$e^{-s \cdot t}=dv$

$\frac{e^{-st}}{-s}=v$

$$= \frac{a}{s} \cdot \lim_{w \to +\infty} \cos at \cdot \frac{e^{-st}}{-s} \Bigg|_{t=0}^{t=w} - \frac{a}{s} \int_{t=0}^{t=w} \sin at \, dt$$

$$= \frac{a}{s} \cdot \lim_{w \to +\infty} \left(\cos(aw) \cdot \frac{e^{-s \cdot w}}{-s} - \cos a \cdot 0 \; \frac{e^{-s \cdot 0}}{-s} - \frac{a}{s} \int_0^w e^{-st} \sin at \, dt \right.$$

$$= \frac{a}{s} \lim_{w \to +\infty} \cos aw \cdot \frac{e^{-s \cdot w}}{-s} - \cos(a \cdot 0) \cdot \frac{e^{-s \cdot 0}}{-s} - \frac{a}{s} \int_0^w e^{-st} \cdot \sin^{at} dt$$

$$= \frac{a}{s} \lim_{w \to +\infty} \left(\frac{1}{s} - \frac{a}{s} \int_0^w e^{-st} \sin at \, dt \right)$$

$$L(\sin at)\left(1 + \frac{a^2}{s^2}\right) = \frac{a}{s^2}$$

$$L(\sin at) = \frac{\frac{a}{\cancel{s^2}}}{\frac{a^2+s^2}{\cancel{s^2}}} = \frac{a}{a^2+s^2} \quad , \quad L\left(e^{2t} + 2 + \sin 3t\right)$$

$$= \frac{1}{s-2} + \frac{2}{s} + \frac{3}{s^2+9}$$

$f(t)$	$L(f(t)) = F(s)$	
① c	$\frac{c}{s}$	$s>0$ ✓
② $e^{\alpha \cdot t}$	$\frac{1}{s-\alpha}$	$\alpha>0$
③ t^n	$\frac{n!}{s^{n+1}}$	
④ $\sin at$	$\frac{a}{a^2+s^2}$	
⑤ $\cos at$	$\frac{s}{a^2+s^2}$	
⑥ $e^{\alpha \cdot t} \cdot f(t)$	$F(s-\alpha)$	
⑦ $e^{\alpha \cdot t} \cdot \sin at$	$\frac{a}{a^2+(s-\alpha)^2}$	
⑧ $e^{\alpha \cdot t} \cdot \cos at$	$\frac{s-\alpha}{a^2+(s-\alpha)^2}$	

(1) / $L^{-1}\left(\dfrac{3}{s^2-3s+2}\right) = ?$

$$\frac{3}{s^2-3s+2} = \frac{A}{(s-2)} + \frac{B}{(s-1)}$$

$(s-1)$ $(s-2)$

$$3 = s\underbrace{(A+B)}_{0} \underbrace{-A-2B}_{-3}$$

$$\cancel{A}+B=0$$
$$-\cancel{A}-2B=-3$$

$-B=-3$ $B=3$

$A=-3$

$$= L^{-1}\left(\frac{-3}{s-2}+\frac{3}{s-1}\right) = L^{-1}\left(\frac{-3}{s-2}\right) + L^{-1}\left(\frac{3}{s-1}\right)$$

$$-3L^{-1}\left(\frac{1}{s-\underset{\alpha}{2}}\right) + 3L^{-1}\left(\frac{1}{s-\underset{\alpha}{1}}\right)$$

$$= -3.e^{2t} + 3e^{t}$$

(2) / $L^{-1}\left(\dfrac{2s+3}{s^2+4}\right) \rightarrow L^{-1}\left(\dfrac{2s}{s^2+4}\right) + L^{-1}\left(\dfrac{3}{s^2+4}\right)$

$$= 2L^{-1}\left(\frac{s}{s^2+2^2}\right) + \frac{3}{2}L^{-1}\left(\frac{\cancel{1}\,2}{s^2+2^2}\right)$$

$$= 2\cos 2t + \frac{3}{2}\sin 2t.$$

$$= \frac{1}{2}L^{-1}\left(\frac{2}{s^2+2^2}\right) = \frac{1}{2}\sin 2t \rightarrow ?.$$

4.2) $\mathcal{L}^{-1}\left(\dfrac{3s-5}{4s^2-4s+37}\right) = ?$

$\Delta = (-4)^2 - 4\cdot 4\cdot 37 < 0$

$4\left(s^2 - s + \dfrac{37}{4}\right)$

$s^2 - s + \left(\dfrac{1}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2 + \dfrac{37}{4} \Rightarrow \left(s-\dfrac{1}{2}\right)^2 - \dfrac{1}{4} + \dfrac{37}{4}$

$= \left(s-\dfrac{1}{2}\right)^2 + 9 \longrightarrow$

Benzetecegiz

$3s-5 = 3\left(s-\dfrac{5}{3}\right)$

$= 3\left(s - \dfrac{1}{2} + \underset{(3)}{\dfrac{1}{2}} - \underset{(2)}{\dfrac{5}{3}}\right)$

$= 3\left(s - \dfrac{1}{2} - \dfrac{7}{6}\right)$

$= 3\left(s-\dfrac{1}{2}\right) - \dfrac{7}{2}$

$\mathcal{L}^{-1}\left(\dfrac{3\left(s-\frac{1}{2}\right) - \frac{7}{2}}{4\left(s-\frac{1}{2}\right)^2 + 3^2}\right) = \dfrac{3}{4}\,\mathcal{L}^{-1}\left(\dfrac{s-\frac{1}{2}}{\left(s-\frac{1}{2}\right)^2+3^2}\right) - \dfrac{\frac{7}{2}}{3\cdot 4}\cdot\mathcal{L}^{-1}\left(\dfrac{3}{\left(s-\frac{1}{2}\right)^2+3^2}\right)$

$= \dfrac{3}{4}\cos 3t\cdot e^{1/2\cdot t} - \dfrac{7}{24}\sin 3t\, e^{1/2\cdot t}$

* $\mathcal{L}\left(\sin 3t\cdot e^{6t}\right) = \dfrac{3}{9+s^2}\Bigg|_{s\to s-6} = \dfrac{3}{9+(s-6)^2}$

* $\mathcal{L}\left(\cos 2t\cdot e^{-t}\right) = \dfrac{s}{s^2+2^2}\Bigg|_{s\to s-\alpha = s+1} = \dfrac{s+1}{(s+1)^2+2^2}$

* $\mathcal{L}\left(t^3\cdot e^{-3t}\right) = \dfrac{3!}{s^4}\Bigg|_{s\to s+3} = \dfrac{6}{(s+3)^4}$

Uygulama:

1) $y=e^{-x}$, $y=e^{x}$ ve $x=1$ doğrusu arasında kalan bölgenin alanı?

$y=e^{-x} \rightarrow \dfrac{1}{e^{x}}$, $x=0 \quad y=1$

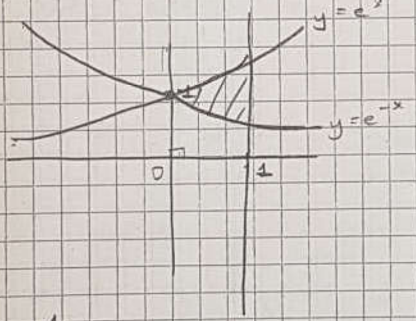
$x=-\infty \quad y=+\infty$

$x=+\infty \quad y=0$

$y=e^{x} \rightarrow x=0 \quad y=1$

$x=+\infty \quad y=+\infty$

$x=-\infty \quad y=0$

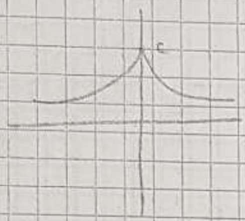


$$A=\int_0^1 (e^{x}-e^{-x})\,dy = e^{y} - \frac{e^{-x}}{-1}\Big|_0^1$$

$$= e^{x}+e^{-y}\Big|_0^1 = e+e^{-1}-2.$$

a) $y = \frac{-4}{1+x^2}$ ve $y = 2-x^2$ eğrileri arasında kalan bölgenin alanını bulunuz

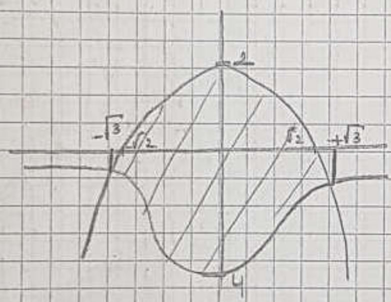
$y = \frac{c}{1+x^2}$ $c>0$



$y = \frac{c}{1+x^2}$ $c<0$

$y = 2-x^2$ için $x=0$ $y=2$

$y=0$ $x=\mp\sqrt{2}$



kesişim

$$\frac{-4}{1+x^2} = 2-x^2$$

$$-4 = 2 - x^2 + 2x^2 - x^4$$

$$x^4 - x^2 - 6 = 0$$

$$(x^2-3)(x^2+2) = 0$$

$x^2 = 3$ $\quad x^2 \neq -2$

$x = \mp\sqrt{3}$ $\quad \cancel{x}$

$$A = \int_{-\sqrt{3}}^{+\sqrt{3}} \left(2-x^2 - \left(\frac{-4}{1+x^2}\right)\right) dx$$

$$A = 2x - \frac{x^3}{3} + 4\arctan x \Big|_{x=-\sqrt{3}}^{x=\sqrt{3}}$$

$$A = \sqrt{3} + 4\frac{\pi}{3} - \left(-\sqrt{3} - 4\frac{\pi}{3}\right)$$

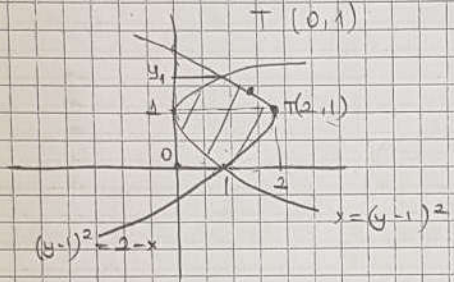
$$A = 2\sqrt{3} + \frac{8\pi}{3}$$

3) $x=(y-1)^2$ ve $(y-1)^2=2-x$ eğrileri arasında kalan böl. alanını hesapla

$$x=(y-1)^2$$
$$\downarrow \quad \downarrow$$
$$0 \quad 1$$

(0 yapan değerlere bakıyoruz.)

$T(0,1)$



$$(y-1)^2=2-x$$
$$\downarrow \qquad \downarrow$$
$$1 \qquad 2$$

$T(2,1)$

$$(y-1)^2=2-(y-1)^2$$
$$2(y-1)^2=2$$
$$(y-1)^2=1$$
$$y-1=\mp 1$$

$y-1=1$ için $y=2$

$y-1=-1$ için $y=0$

$$A=\int_{y=0}^{y=2}\left[2-(y-1)^2-(y-1)^2\right]dy$$

$$=2y-\frac{2(y-1)^3}{3}\Bigg|_0^2$$

$$=2.2-\frac{2}{3}(2-1)^3-\left(2.0-\frac{2}{3}(0-1)^3\right)$$

$$=4-\frac{2}{3}-\frac{2}{3}=4-\frac{4}{3}=\frac{8}{3}$$

x'e göre

2) $\int_0^1 (1+\sqrt{x})-(1-\sqrt{x})\,dx + \int_2^1 1+\sqrt{2-x}-(1-\sqrt{2-x})\,dx$

1) $\sqrt{x}=\sqrt{(y-1)^2}$

$|y-1|=\sqrt{x}$

$y=\mp\sqrt{x}+1$

$y=1+\sqrt{x}$

$y=1-\sqrt{x}$

$\sqrt{(y-1)^2}=\sqrt{2-x}$

$|y-1|=\sqrt{2-x}$

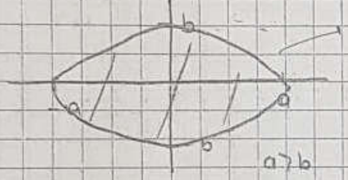
$y-1=\mp\sqrt{2-x}$

$y=1\mp\sqrt{2-x}$

4) $\begin{cases} x = a.\cos t \\ y = b.\sin t \end{cases}$ par. denklemiyle verilen eğrinin sınırladığı bölgenin alanı.

$$\left.\begin{aligned} \frac{x}{a} &= \cos t \\ \frac{y}{b} &= \sin t \end{aligned}\right\} \rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{(Elips)}$$

t-yol: $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $\quad t_1 \le t \le t_2 \quad \displaystyle\int_{t_1}^{t_2} y(t) . x'(t)\, dt$



$0 \le x \le a$

$x = 0$ için $\quad 0 = \cos t$, $\; t = \pi/2$

$a > b \quad x = a$ için $\quad \cos t = 1$, $\; t = 0$

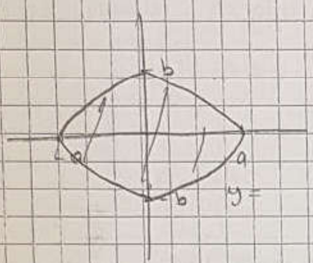
$$\int_{\pi/2}^{0} b.\sin t . -a.\sin t\, dt = ab \int_{0}^{\pi/2} \sin^2 t\, dt$$

$$ab \int_{0}^{\pi/2} \frac{1-\cos 2t}{2}\, dt = ab.\left(\frac{1}{2} - \frac{\sin 2t}{2.2}\right)\Bigg|_{t=0}^{t=\pi/2}$$

$$= ab.\left(\frac{\frac{\pi}{2}}{2} - \sin\frac{2.\frac{\pi}{2}}{4} - 0\right)$$

$\frac{A}{4} = ab\frac{\pi}{4} \qquad A = ab\pi$

2. yol: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=b^2\left(1-\frac{x^2}{a^2}\right)$$

$$|y|=\frac{b}{a}\sqrt{a^2-x^2}$$

$$y=\mp\frac{b}{a}\sqrt{a^2-x^2}$$

$$A=\int_{-a}^{+a}\left(\frac{b}{a}\sqrt{a^2-x^2}+\frac{b}{a}\sqrt{a^2-x^2}\right)dx=\int_{-a}^{a}\frac{2b}{a}\sqrt{a^2-x^2}\,dx$$

$$=\frac{2b}{a}\cdot\int_{-a}^{+a}\sqrt{a^2-x^2}\,dx$$

$$A=\frac{2b}{a}\int_{\alpha}^{\beta}\sqrt{a^2-a^2\sin^2 t}\cdot a\cos t\,dt$$

$$=\frac{2b}{a}\cdot a^2\int_{\alpha}^{\beta}\cos^2 t\,dt=2ab\int_{\alpha}^{\beta}\frac{1+\cos 2t}{2}\,dt=2ab\int_{-\pi/2}^{+\pi/2}\frac{1+\cos 2t}{2}\,dt$$

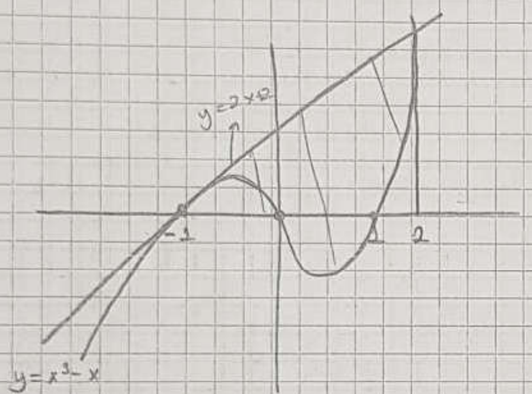
$$A=a.b\left(t+\frac{\sin 2t}{2}\right)\Bigg|_{-\pi/2}^{\pi/2}=ab\left(\frac{\pi}{2}+\frac{\overbrace{\sin\left(2\cdot\pi/2\right)}^{0}}{2}-\frac{-\pi}{2}+\frac{\overbrace{\sin\left(2\cdot\frac{-\pi}{2}\right)}^{0}}{2}\right)$$

$$=ab\cdot\pi$$

1) $y=x^3-x$ eğrisi ile bu eğriye $x=-1$ apsisli nok. teğet olan doğru arasında kalan bölgenin alanını bulunuz.

$y=0$ için $x(x^2-1)=0$

$x_1=-1$ $x_2=0$ $x_3=1$



$x<-1$ için $x=-2$

$y=(-2)^3+2$

$y=-6$ aşağıdan gelecek

$-1<x<0$ için $x=\frac{-1}{2}$

$y=\left(\frac{-1}{2}\right)^3-\frac{1}{2}$

$=\frac{1}{8}-\frac{1}{2}=\frac{3}{8}>0$ yukarıdan gelecek

$0<x<1$ için

$x=\frac{1}{2}$ $y=\left(\frac{1}{2}\right)^3-\frac{1}{2}$

$=\frac{1}{8}-\frac{1}{2}=<0$

$x>1$ için $x=2$

$y=2^3-2$

$=6$

Teğet doğrusu

$y-y_0=m(x-x_0)$

$y'=3x^2-1\Big|_{x=-1}$

$3-1=2 \rightarrow m$

$x_0=-1$ için $(-1)^3-(-1)=0$

$y=2(x+1)$

$y=2x+2$

Ortak çözüm

$2x+2=x^3-x$

$x^3-3x-2=0$

>1 >2 x zaten 1'den büyük olacak

$x=2$

$2^3-3.2-2=8-6-2=0$

sağladı ✓

$$A = \int_{-1}^{2} \left[2x+2 - (x^3 - x)\right] dx$$

$$= \frac{2x^2}{2} + 2x - \frac{x^4}{4} + \frac{x^2}{2} \Big|_{-1}^{2}$$

$$A = \frac{27}{4}$$

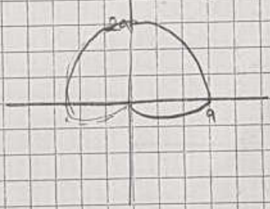
6) $r = a.(1+\sin\theta)$ kardiyodinin içinde $r=a$ çemberinin dışında kalan bölgenin alanını bulunuz.

$r = a(1+\sin\theta) \qquad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

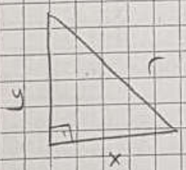
$r = a\left(1+\sin\left(-\frac{\pi}{2}\right)\right) = 0$

$r = a.(1+\sin 0) = a$

$r = a\left(1+\sin\frac{\pi}{2}\right) = 2a$

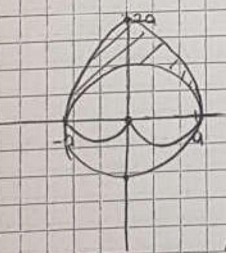


$$\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2(\theta)\, d\theta$$



$r^2 = x^2 + y^2$

$\boxed{x^2+y^2=a^2}$



$$\frac{A}{2} = \frac{1}{2}\int_{0}^{\pi/2} \left[(a(1+\sin\theta))^2 - a^2\right] d\theta$$

$$A = \int_{0}^{\pi/2} a^2(1+2\sin\theta+\sin^2\theta - 1)\, d\theta$$

$$A = a^2\int_{0}^{\pi/2} \left[2\sin\theta + \frac{1-\cos 2\theta}{2}\right] d\theta$$

$$A = a^2\left(-2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{2\cdot 2}\right)\Big|_{\theta=0}^{\theta=\pi/2}$$

$$A = a^2 \left(-2 \cos\frac{\pi}{2} + \frac{\pi/2}{2} - \frac{\sin\left(2 \cdot \frac{\pi}{2}\right)}{4} \right) - \left(-2 \cdot \cos 0 + \frac{0}{2} - \frac{\sin 0}{4} \right)$$

$$A = a^2 \left(\frac{\pi}{4} + 2 \right) \checkmark$$

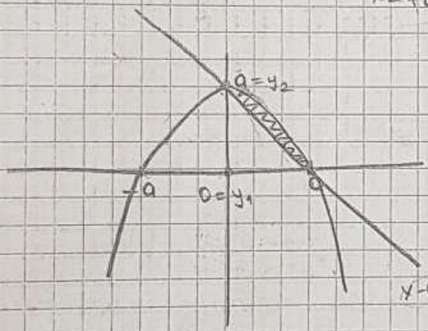
17) $y = a - \frac{x^2}{a}$ parabol eğrisi ile $x + y = a$ doğruları tarafından sınırlanan bölgenin y ekseni etrafında döndürülmesiyle meydana gelen dönel cismin hacmini bulunuz.

$x = 0$ için $y = a$ | $x + y = a$

$y = 0$ için $\frac{x^2}{a} = a$ | $x = 0 \quad y = a \quad (0, a)$

$\sqrt{x^2} = \sqrt{a^2}$ | $y = 0 \quad x = a \quad (a, 0)$

$x = \mp a$



$$V = \pi \cdot \int_{x_1}^{x_2} f^2(x)\, dx \quad \text{veya}$$

$$V = \pi \int_{y_1}^{y_2} f^2(y)\, dy$$

$$y - a = \frac{-x^2}{a}$$

$$\sqrt{a(a-y)} = \sqrt{x^2} \qquad y - a =$$

$$x = \mp \sqrt{a(a-y)} \qquad \frac{x^2}{a} = a - y$$

$$x^2 = a \cdot (a - y)$$

$$x = \mp \sqrt{a \cdot (a-y)}$$

$$V = \pi \int_0^a \left[\left(\sqrt{a \cdot (a-y)}\right)^2 - (a-y)^2 \right] dy$$

$$= \pi \int_0^a \left(a(a-y) - (a-y)^2 \right) dy = \pi \int_0^a \left(\cancel{a^2} - ay - \cancel{a^2} + 2ay - y^2 \right) dy$$

$$= \pi \int_0^a (ay - y^2)\, dy = \pi \left(a\frac{y^2}{2} - \frac{y^3}{3} \right) \Bigg|_{y=0}^{y=a}$$

$$= \pi \cdot \left(\frac{a^2 \cdot a}{2} - \frac{a^3}{3} \right) = \pi \left(\frac{a^3}{2} - \frac{a^3}{3} \right)$$

$$V = \frac{a^3 \pi}{6}$$

8) Parametrik denklemi $\begin{cases} x = \cos t + \ln\left(\tan \frac{t}{2}\right) \\ y = \sin t \end{cases}$

olan eğrinin $t_0 = \pi/4$ $t_1 = \pi/3$ nok. arasındaki eğri parça. uzunluğunu hesaplayın.

1) $y = f(x)$ $\quad \displaystyle\int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}\, dx$ kartezyen

2) $x = g(y)$ $\quad \displaystyle \ell = \int_{x_1}^{x_2} \sqrt{1 + (g'(y))^2}\, dy$

3) parametrik $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $\quad t_1 \leq t \leq t_2 \quad \displaystyle \ell = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

4) $r = r\vartheta$ $\quad \ell = \sqrt{r^2(\vartheta) . (r'(\vartheta))^2}\, d\vartheta$ kutupsal

$$y'(t) = -\sin t + \frac{\frac{1}{2} \cdot \frac{1}{\cos^2 \frac{t}{2}}}{\frac{\sin t/2}{\cos t/2}} \rightarrow -\sin t + \frac{1}{\underbrace{2 \cos \frac{t}{2} \sin \frac{t}{2}}}$$

$$-\sin t + \frac{1}{\sin t} = \frac{\cos^2 t}{\sin t} \qquad y'(t) = \cos t$$

$$\frac{\cos^4 t}{\sin^2 t} + \cos^2 t = \frac{\cos^4 t + \cos^2 t \cdot \sin^2 t}{\sin^2 t}$$

$$\frac{\cos^2 t \left(\cos^2 t + \sin^2 t\right)}{\sin^2 t} = \frac{\cos^2 t}{\sin^2 t}$$

$$\ell = \int_{\pi/4}^{\pi/3} \frac{\cos t\, dt}{\sin t} = \int_{\alpha}^{\beta} \frac{dw}{w} \rightarrow \ln|w| \Big|_{\alpha}^{\beta}$$

$\sin t = w$
$\cos t\, dt = dw$

$$= \ln(\sin t) \Big|_{\pi/4}^{\pi/3}$$

$$\ell = \ln|\sin \pi/3| - \ln|\sin \pi/4|$$

$$= \ln\left|\frac{\sqrt{3}}{2}\right| - \ln\left|\frac{\sqrt{2}}{2}\right| = \ln\left|\frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{\sqrt{2}}{\cancel{2}}}\right|$$

$$\ln\left|\frac{\sqrt{3}}{\sqrt{2}}\right|$$

9) $\int_0^{+\infty} x \cdot e^{-x^2}\, dx = ?$

$f(x) = x \cdot e^{-x^2} \in [0, +\infty)$ 1. tip Gen. int.

$\lim_{u \to +\infty} \boxed{\int_0^u x \cdot e^{-x^2} dx}$ Belirli int.

$\lim_{u \to +\infty} \int_\alpha^\beta e^{-u} \frac{1}{2} du = \frac{1}{2} \lim_{u \to +\infty} (-e^{-u}) \Big|_\alpha^\beta$

$x^2 = u$
$2x\,dx = du$
$x\,dx = \frac{du}{2}$

$= \frac{1}{2} \lim_{u \to +\infty} \left(-e^{-x^2}\right) \Big|_0^u$

$= \frac{1}{2} \lim_{u \to +\infty} -e^{-u^2} - (-e^0)$

$= \frac{1}{2} \lim_{u \to +\infty} \left(\frac{-1}{\cancel{e^{u^2}}} + 1\right) = \frac{1}{2}$

10) $\int_0^1 \frac{\arcsin x}{\sqrt{1-x^2}} dx = ?$

$1 - x^2 = 0$
$x = \mp 1$

$x = 1 \in [0,1]$ old. $x=1$ süreksizlik nokt.

$\lim_{b \to 0^+} \int_0^{1-b}$ $\lim_{b \to 0^-} \int_0^{1+b}$

$\arcsin x = u$

$\frac{1}{\sqrt{1-x^2}} dx = du$

$b \to 0^+$, $1-b$

$I = \lim_{b \to 0^+} \boxed{\int_0^{1-b} \frac{\arcsin x}{\sqrt{1-x^2}} dx}$ Belirli int.

$\lim_{b \to 0^+} \int_\alpha^\beta u \cdot du = \lim_{b \to 0^+} \frac{u^2}{2} \Big|_\alpha^\beta$

$\lim_{b \to 0^+} \frac{(\arcsin x)^2}{2} \Big|_0^{1-b}$

$\lim_{b \to 0^+} \frac{(\arcsin(1-b))^2}{2} - \cancel{\frac{(\arcsin(0))^2}{2}}$

$\frac{\pi^2}{8}$

1) : $y = x^3$ eğrisi ile bu eğriye (1,1) nok. çizilen teğetin arasında kalan bölgenin alanı? $\rightarrow \frac{27}{4}$

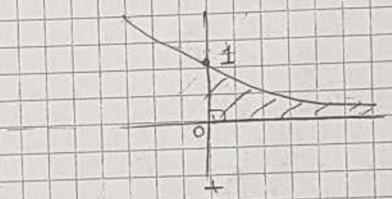
2) - 1. bölgede $y = e^{-x}$ eğrisi ile x eks. arasında kalan bölgenin,

a) Alanı?
b) x eks. etrafında dön. ile meydana gelen dönel cismin hacmi?

$y = e^{-x}$ $x = 0$ için $y = 1$

$x = +\infty$ $y = 0$

$x = -\infty$ $y = +\infty$



a) $A = \int_0^{+\infty} e^{-x}\,dx \rightarrow \lim_{t \to +\infty} \int_0^t e^{-x}\,dx \rightarrow \lim_{t \to +\infty} -\int_0^t e^{-x} \rightarrow \lim_{t \to +\infty} -e^{-x}\Big|_0^t$

$\lim_{t \to +\infty} -e^{-t} + 1 \Big|_0^t$

$= 1$

b) $V = \pi \int_0^{+\infty} \left[(e^{-x})^2 - 0^2 \right] dx \Rightarrow \pi \int_0^{\infty} e^{-2x} \rightarrow \lim_{t \to +\infty} -\pi \cdot \frac{e^{-2x}}{2} \Big|_0^t$

$\lim_{t \to +\infty} \frac{-\pi \cdot \cancel{e^{-2t}}}{\cancel{2}} + \frac{\pi}{2} = \frac{\pi}{2}$ ✓

$L^{-1}\left(\frac{2s+3}{s^2+4} \right) = ?$

$\frac{s}{a^2+s^2} \rightarrow \cos at$

$\frac{a}{s^2+a^2} = \sin at$

$L^{-1}\left(\frac{2s}{s^2+4} \right)_{a=2} + \frac{L^{-1}}{2}\left(\frac{3 \cdot 2}{s^2+4} \right)_{a=2}$

$2 \cdot \cos 2t + \frac{3}{2} \cdot \sin 2t$