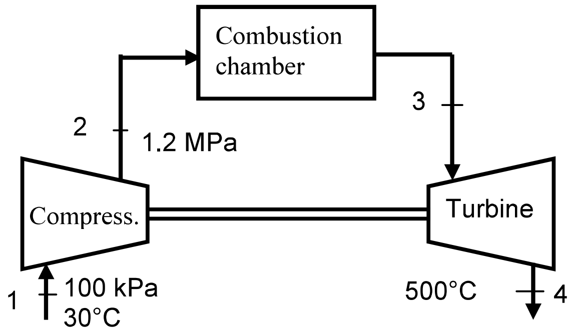
9.80

9-84 A gas-turbine plant operates on the simple Brayton cycle. The net power output, the back work ratio, and the thermal efficiency are to be determined.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with variable specific heats.

Properties The gas constant of air is R = 0.287 kJ/kg·K (Table A-1).

Analysis (a) For this problem, we use the properties from EES software. Remember that for an ideal gas, enthalpy is a function of temperature only whereas entropy is functions of both temperature and pressure.



Process 1-2: Compression

$$T_1 = 30°\text{C} \longrightarrow h_1 = 303.60 \text{ kJ/kg}$$

$$\left.\begin{array}{l} T_1 = 30°\text{C} \\ P_1 = 100 \text{ kPa} \end{array}\right\} s_1 = 5.7159 \text{ kJ/kg} \cdot \text{K}$$

$$\left.\begin{array}{l} P_2 = 1200 \text{ kPa} \\ s_2 = s_1 = 5.7159 \text{ kJ/kg.K} \end{array}\right\} h_{2s} = 617.37 \text{ kJ/kg}$$

$$\eta_\text{C} = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow 0.82 = \frac{617.37 - 303.60}{h_2 - 303.60} \longrightarrow h_2 = 686.24 \text{ kJ/kg}$$

Process 3-4: Expansion

$$T_4 = 500°\text{C} \longrightarrow h_4 = 792.62 \text{ kJ/kg}$$

$$\eta_\text{T} = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow 0.88 = \frac{h_3 - 792.62}{h_3 - h_{4s}}$$

We cannot find the enthalpy at state 3 directly. However, using the following lines in EES together with the isentropic efficiency relation, we find h_3 = 1404.7 kJ/kg, T_3 = 1034ºC, s_3 = 6.5699 kJ/kg.K. The solution by hand would require a trial-error approach.

```
h_3=enthalpy(Air, T=T_3)
s_3=entropy(Air, T=T_3, P=P_2)
h_4s=enthalpy(Air, P=P_1, s=s_3)
```

The mass flow rate is determined from

$$\dot{m} = \frac{P_1 \dot{V}_1}{RT_1} = \frac{(100 \text{ kPa})(150/60 \text{ m}^3\text{/s})}{(0.287 \text{ kPa} \cdot \text{m}^3\text{/kg} \cdot \text{K})(30 + 273 \text{ K})} = 2.875 \text{ kg/s}$$

The net power output is

$$\dot{W}_\text{C,in} = \dot{m}(h_2 - h_1) = (2.875 \text{ kg/s})(686.24 - 303.60)\text{kJ/kg} = 1100 \text{ kW}$$

$$\dot{W}_\text{T,out} = \dot{m}(h_3 - h_4) = (2.875 \text{ kg/s})(1404.7 - 792.62)\text{kJ/kg} = 1759 \text{ kW}$$

$$\dot{W}_\text{net} = \dot{W}_\text{T,out} - \dot{W}_\text{C,in} = 1759 - 1100 = \mathbf{659\ kW}$$

(b) The back work ratio is

$$r_\text{bw} = \frac{\dot{W}_\text{C,in}}{\dot{W}_\text{T,out}} = \frac{1100 \text{ kW}}{1759 \text{ kW}} = \mathbf{0.625}$$

(c) The rate of heat input and the thermal efficiency are

$$\dot{Q}_\text{in} = \dot{m}(h_3 - h_2) = (2.875 \text{ kg/s})(1404.7 - 686.24)\text{kJ/kg} = 2065 \text{ kW}$$

$$\eta_{th} = \frac{\dot{W}_\text{net}}{\dot{Q}_\text{in}} = \frac{659 \text{ kW}}{2065 \text{ kW}} = \mathbf{0.319}$$

9.89

9-93 An ideal Brayton cycle with regeneration is considered. The effectiveness of the regenerator is 100%. The net work output and the thermal efficiency of the cycle are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

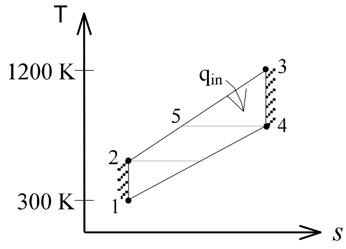
Analysis Noting that this is an ideal cycle and thus the compression and expansion processes are isentropic, we have

$$T_1 = 300 \text{ K} \longrightarrow \begin{array}{l} h_1 = 300.19 \text{ kJ/kg} \\ P_{r_1} = 1.386 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (10)(1.386) = 13.86 \longrightarrow h_2 = 579.87 \text{ kJ/kg}$$

$$T_3 = 1200 \text{ K} \longrightarrow \begin{array}{l} h_3 = 1277.79 \text{ kJ/kg} \\ P_{r_3} = 238 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{10}\right)(238) = 23.8 \longrightarrow h_4 = 675.85 \text{ kJ/kg}$$



$$w_{\text{C,in}} = h_2 - h_1 = 579.87 - 300.19 = 279.68 \text{ kJ/kg}$$

$$w_{\text{T,out}} = h_3 - h_4 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

Thus,

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 601.94 - 279.68 = \mathbf{322.26 \text{ kJ/kg}}$$

Also, $\varepsilon = 100\% \longrightarrow h_5 = h_4 = 675.85 \text{ kJ/kg}$

$$q_{\text{in}} = h_3 - h_5 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{322.26 \text{ kJ/kg}}{601.94 \text{ kJ/kg}} = \mathbf{53.5\%}$$

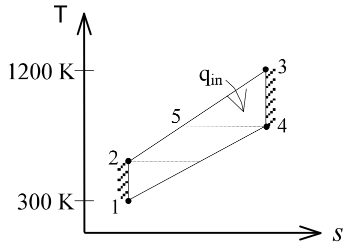
9.90

9-95 An ideal Brayton cycle with regeneration is considered. The effectiveness of the regenerator is 100%. The net work output and the thermal efficiency of the cycle are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with constant specific heats at room temperature. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a).

Analysis Noting that this is an ideal cycle and thus the compression and expansion processes are isentropic, we have



$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{ K})(10)^{0.4/1.4} = 579.2\text{ K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1200\text{ K})\left(\frac{1}{10}\right)^{0.4/1.4} = 621.5\text{ K}$$

$$\varepsilon = 100\% \longrightarrow T_5 = T_4 = 621.5\text{ K and } T_6 = T_2 = 579.2\text{ K}$$

$$\eta_{\text{th}} = 1-\frac{q_{\text{out}}}{q_{\text{in}}} = 1-\frac{c_p(T_6-T_1)}{c_p(T_3-T_5)} = 1-\frac{T_6-T_1}{T_3-T_5} = 1-\frac{579.2-300}{1200-621.5} = \mathbf{0.517}$$

or $$\eta_{\text{th}} = 1-\left(\frac{T_1}{T_3}\right)r_p^{(k-1)/k} = 1-\left(\frac{300}{1200}\right)(10)^{(1.4-1)/1.4} = 0.517$$

Then,

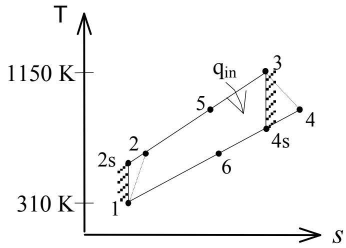
$$\begin{aligned} w_{\text{net}} &= w_{\text{turb, out}} - w_{\text{comp, in}} = (h_3-h_4)-(h_2-h_1) \\ &= c_p[(T_3-T_4)-(T_2-T_1)] \\ &= (1.005\text{ kJ/kg.K})[(1200-621.5)-(579.2-300)]\text{K} \\ &= \mathbf{300.8\ kJ/kg} \end{aligned}$$

or,

$$\begin{aligned} w_{\text{net}} &= \eta_{\text{th}} q_{\text{in}} \\ &= \eta_{\text{th}}(h_3-h_5) \\ &= \eta_{\text{th}} c_p (T_3-T_5) \\ &= (0.517)(1.005\text{ kJ/kg.K})(1200-621.5) \\ &= 300.6\text{ kJ/kg} \end{aligned}$$

9-92. **9-96** A Brayton cycle with regeneration using air as the working fluid is considered. The air temperature at the turbine exit, the net work output, and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.



Properties The properties of air are given in Table A-17.

Analysis (a) The properties of air at various states are

$$T_1 = 310 \text{ K} \longrightarrow \begin{array}{l} h_1 = 310.24 \text{ kJ/kg} \\ P_{r_1} = 1.5546 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (7)(1.5546) = 10.88 \longrightarrow h_{2s} = 541.26 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_C = 310.24 + (541.26 - 310.24)/(0.75) = 618.26 \text{ kJ/kg}$$

$$T_3 = 1150 \text{ K} \longrightarrow \begin{array}{l} h_3 = 1219.25 \text{ kJ/kg} \\ P_{r_3} = 200.15 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{7}\right)(200.15) = 28.59 \longrightarrow h_{4s} = 711.80 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s}) = 1219.25 - (0.82)(1219.25 - 711.80) = 803.14 \text{ kJ/kg}$$

Thus, $T_4 =$ **782.8 K**

(*b*)
$$\begin{aligned} w_{\text{net}} &= w_{\text{T,out}} - w_{\text{C,in}} = (h_3 - h_4) - (h_2 - h_1) \\ &= (1219.25 - 803.14) - (618.26 - 310.24) \\ &= \mathbf{108.09 \text{ kJ/kg}} \end{aligned}$$

(*c*)
$$\varepsilon = \frac{h_5 - h_2}{h_4 - h_2} \longrightarrow \begin{aligned} h_5 &= h_2 + \varepsilon(h_4 - h_2) \\ &= 618.26 + (0.65)(803.14 - 618.26) \\ &= 738.43 \text{ kJ/kg} \end{aligned}$$

Then,

$$q_{\text{in}} = h_3 - h_5 = 1219.25 - 738.43 = 480.82 \text{ kJ/kg}$$

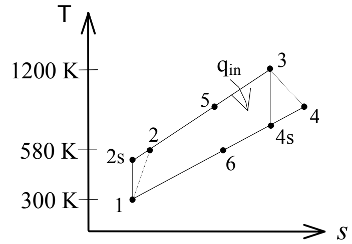
$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{108.09 \text{ kJ/kg}}{480.82 \text{ kJ/kg}} = \mathbf{22.5\%}$$

9-98 A regenerative gas-turbine engine using air as the working fluid is considered. The amount of heat transfer in the regenerator and the thermal efficiency are to be determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis (*a*) The properties at various states are



$$r_p = P_2 / P_1 = 800/100 = 8$$

$$T_1 = 300 \text{ K} \longrightarrow h_1 = 300.19 \text{ kJ/kg}$$
$$T_2 = 580 \text{ K} \longrightarrow h_2 = 586.04 \text{ kJ/kg}$$
$$T_3 = 1200 \text{ K} \longrightarrow h_3 = 1277.79 \text{ kJ/kg}$$
$$P_{r_3} = 238.0$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(238.0) = 29.75 \longrightarrow h_{4s} = 719.75 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow h_4 = h_3 - \eta_T (h_3 - h_{4s})$$
$$= 1277.79 - (0.86)(1277.79 - 719.75)$$
$$= 797.88 \text{ kJ/kg}$$

$$q_{\text{regen}} = \varepsilon (h_4 - h_2) = (0.72)(797.88 - 586.04) = \mathbf{152.5 \text{ kJ/kg}}$$

(*b*)
$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = (h_3 - h_4) - (h_2 - h_1)$$
$$= (1277.79 - 797.88) - (586.04 - 300.19) = 194.06 \text{ kJ/kg}$$

$$q_{\text{in}} = (h_3 - h_2) - q_{\text{regen}} = (1277.79 - 586.04) - 152.52 = 539.23 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{194.06 \text{ kJ/kg}}{539.23 \text{ kJ/kg}} = \mathbf{36.0\%}$$

9.104

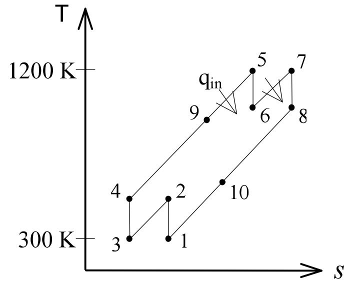
9-108 An ideal gas-turbine cycle with two stages of compression and two stages of expansion is considered. The back work ratio and the thermal efficiency of the cycle are to be determined for the cases of with and without a regenerator.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis (*a*) The work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine since this is an ideal cycle. Then,

$$T_1 = 300 \text{ K} \longrightarrow \begin{array}{l} h_1 = 300.19 \text{ kJ/kg} \\ P_{r_1} = 1.386 \end{array}$$



$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (3)(1.386) = 4.158 \longrightarrow h_2 = h_4 = 411.26 \text{ kJ/kg}$$

$$T_5 = 1200 \text{ K} \longrightarrow \begin{array}{l} h_5 = h_7 = 1277.79 \text{ kJ/kg} \\ P_{r_5} = 238 \end{array}$$

$$P_{r_6} = \frac{P_6}{P_5} P_{r_5} = \left(\frac{1}{3}\right)(238) = 79.33 \longrightarrow h_6 = h_8 = 946.36 \text{ kJ/kg}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}$$

Thus, $$r_{\text{bw}} = \frac{w_{\text{C,in}}}{w_{\text{T,out}}} = \frac{222.14 \text{ kJ/kg}}{662.86 \text{ kJ/kg}} = \mathbf{33.5\%}$$

$$q_{\text{in}} = (h_5 - h_4) + (h_7 - h_6) = (1277.79 - 411.26) + (1277.79 - 946.36) = 1197.96 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{440.72 \text{ kJ/kg}}{1197.96 \text{ kJ/kg}} = \mathbf{36.8\%}$$

(*b*) When a regenerator is used, r_{bw} remains the same. The thermal efficiency in this case becomes

$$q_{\text{regen}} = \varepsilon(h_8 - h_4) = (0.75)(946.36 - 411.26) = 401.33 \text{ kJ/kg}$$

$$q_{\text{in}} = q_{\text{in,old}} - q_{\text{regen}} = 1197.96 - 401.33 = 796.63 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{440.72 \text{ kJ/kg}}{796.63 \text{ kJ/kg}} = \mathbf{55.3\%}$$

9-110 A regenerative gas-turbine cycle with two stages of compression and two stages of expansion is considered. The minimum mass flow rate of air needed to develop a specified net power output is to be
9.106 determined.

Assumptions **1** The air standard assumptions are applicable. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energy changes are negligible.

Properties The properties of air are given in Table A-17.

Analysis The mass flow rate will be a minimum when the cycle is ideal. That is, the turbine and the compressors are isentropic, the regenerator has an effectiveness of 100%, and the compression ratios across each compression or expansion stage are identical. In our case it is $r_p = \sqrt{9} = 3$. Then the work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine.

$$T_1 = 300 \text{ K} \longrightarrow h_1 = 300.19 \text{ kJ/kg}, \quad P_{r_1} = 1.386$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (3)(1.386) = 4.158 \longrightarrow h_2 = h_4 = 411.26 \text{ kJ/kg}$$

$$T_5 = 1200 \text{ K} \longrightarrow h_5 = h_7 = 1277.79 \text{ kJ/kg}, \quad P_{r_5} = 238$$

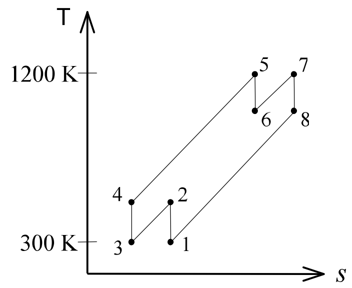
$$P_{r_6} = \frac{P_6}{P_5} P_{r_5} = \left(\frac{1}{3}\right)(238) = 79.33 \longrightarrow h_6 = h_8 = 946.36 \text{ kJ/kg}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2(411.26 - 300.19) = 222.14 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_5 - h_6) = 2(1277.79 - 946.36) = 662.86 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 662.86 - 222.14 = 440.72 \text{ kJ/kg}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{110{,}000 \text{ kJ/s}}{440.72 \text{ kJ/kg}} = \mathbf{249.6 \text{ kg/s}}$$



9-111 A regenerative gas-turbine cycle with two stages of compression and two stages of expansion is considered. The minimum mass flow rate of air needed to develop a specified net power output is to be determined.

Assumptions **1** Argon is an ideal gas with constant specific heats. **2** Kinetic and potential energy changes are negligible.

Properties The properties of argon at room temperature are c_p = 0.5203 kJ/kg.K and k = 1.667 (Table A-2a).

Analysis The mass flow rate will be a minimum when the cycle is ideal. That is, the turbine and the compressors are isentropic, the regenerator has an effectiveness of 100%, and the compression ratios across each compression or expansion stage are identical. In our case it is $r_p = \sqrt{9} = 3$. Then the work inputs to each stage of compressor are identical, so are the work outputs of each stage of the turbine.

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300 \text{ K})(3)^{0.667/1.667} = 465.6 \text{ K}$$

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{(k-1)/k} = (1200 \text{ K})\left(\frac{1}{3}\right)^{0.667/1.667} = 773.2 \text{ K}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2c_p(T_2 - T_1) = 2(0.5203 \text{ kJ/kg}\cdot\text{K})(465.6 - 300)\text{K} = 172.3 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_5 - h_6) = 2c_p(T_5 - T_6) = 2(0.5203 \text{ kJ/kg}\cdot\text{K})(1200 - 773.2)\text{K} = 444.1 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 444.1 - 172.3 = 271.8 \text{ kJ/kg}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{110{,}000 \text{ kJ/s}}{271.8 \text{ kJ/kg}} = \mathbf{404.7 \text{ kg/s}}$$